Coincident Point Theorem and Common Fixed-Point Theorems in M-Fuzzy Metric Spaces

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Abstract

The usefulness of the stability result is shown via the use of an example. Through using the characteristics of -distance mappings and -admissible mappings, we present the idea of generalized contraction mappings and show the existence of a fixed-point theorem for such mappings. This is accomplished by mapping properties. In addition, we extend our conclusion to the theorems of coincidence point and common fixed point in metric spaces. Further, the fixed-point theorems that are endowed with an arbitrary binary relation may also be deduced from our conclusions thanks to this line of reasoning. The theory of fixed points is an intriguing topic that has a huge number of applications in many subfields of mathematics. We have never been able to discover a book that treats the argument ina unitary manner (unless it is particularly dedicated to fixed points), and this may be because of the transversal nature of the topic. In the majority of instances, WE saw that the fixed points appeared whenit was necessary for them to do so. On the other hand, we think that they should merit a meaningful position in any general textbook, and in particular, in a textbook on functional analysis. The primary motivation for my decision to jot down my thoughts was found in the previous sentence. We made an effort to compile the majority of the relevant findings from the topic, after which we discussed a variety of applications that are connected to it.

Keywords: Theorem, Fixed Point, contraction mappings

Introduction

In 1965, Zadeh presented the renowned hypothesis of fuzzy sets and involved it as an instrument for managing vulnerability emerging out of absence of data about specific complex framework. From that point forward, to involve this idea in geography and investigation many creators have expansively fostered the hypothesis of fuzzy sets and applications Fixed point theorems in fuzzy math are arising with vivacious expectation and imperative trust. Apparently Kramosil and Michalek's investigation of fuzzy metric spaces clears a way for extremely alleviating apparatus to foster fixed point theorems for contractive sort maps. George and Veeramany, Kramosil and Michalek have presented the idea of fuzzy topological space prompts by fuzzy metric space which have vital application in quantum molecule material science especially in associations with both string and ε (∞) hypothesis which were given and concentrated by E. I. Naschie. Many creators have made fixed statement hypothesis in fuzzy (probabilistic) metric spaces. Vashuki got the fuzzy adaptation of normal fixed point hypothesis for utilizing additional circumstances. That's what we remember whether the distance between objects is fuzzy, then the article could possibly be fuzzy. That is in fuzzy metric space the set will be fuzzy, yet in fuzzy 2-metric space the distance between objects concerning the closeness capacity will be fuzzy, whilethe set might be fuzzy. The fascinating outcomes with regards to this heading are come from a progression of papers by Gahler, who researched 2-metric space Sharma, Sharma and Iseki concentrates on whenever compression first sort mappings in 2-metric space. As of late Wenzhi and others started theinvestigation of 2-PM spaces. There has been various speculation of metric spaces. One such speculation is generalized metric space on D-metric space started by Dhage in 1992. He made a few guidelines fixed statements for a self- guide fulfilling a compression for complete and limited D-metric spaces. Rhoades generalized Dhage's contractive condition by expanding the quantity of variables and demonstrated the presence of remarkable fixed point of a self-map in D-metric space.

As of late Sedghi and Shobhe presented D*metric space as a likely change of the definition of D□metric essential properties in D*□metric spaces. Utilizing presented by Dhage, and demonstrate a few

D*metric ideas, Sedghi and Shobhe characterize M-fuzzy metric space and made a typical fixed statement hypothesis in it.

OBJECTIVES OF THE STUDY

1.To study on g-metric space by using clrg property

2.The markov-kakutani theorem and schauder-tychonoff fixed point theorem

Some Fixed Point And Coincident Point Theorem In Generalized M - Fuzzy Metric Spaces Theorem Let (X, M, *) be a generalized M - fuzzy metric space and T: X→ X be a planning with the end goal that for all $x \neq y \neq z \in X$ and $t > 0$.

$$\mathcal{M}(Tx, Ty, Tz, t) > \min\{\mathcal{M}(x, y, z, t), \mathcal{M}(x, Tx, Ty, t), \mathcal{M}(z, Ty, Tz, t)\}$$

for any point $x0 \in X$ to such an extent that arrangement $\{T^n(x0)\}$ has an aftereffect unites to u. Then, atthat point, u is interesting fixed point of T.

Evidence:

Let $x0 \in X$ be any erratic fixed component in X.

$x_2 \in X$ such that $x_2 = Tx_1 = T^2x_0$ Then there exists x1 ∈X with the end goal that x1 = Tx0 Comparably there exists

$x_n = T^nx_0$ for all $n \geq 1$ in X. Continuing in this way get a sequence Suppose $x_n = x_{n+1}$ for some n.

Then $x_n = Tx_n$, Thus $x_n = u$ is a fixed point of T.Let us assume that $x_n \neq x_{n+1}$ for all n

For n ≥ 1, We have

$$\mathcal{M}(x_n, x_n, x_{n+1}, t) = \mathcal{M}(Tx_{n-1}, Tx_{n-1}, Tx_n, t)$$

$$> \min\{\mathcal{M}(x_{n-1}, x_{n-1}, x_n, t), \mathcal{M}(x_{n-1}, Tx_{n-1}, Tx_{n-1}, t),$$

$$\mathcal{M}(x_n, Tx_{n-1}, Tx_n, t)\}.$$

$$= \min\{\mathcal{M}(x_{n-1}, x_{n-1}, x_n, t), \mathcal{M}(x_{n-1}, x_n, x_n, t), \mathcal{M}(x_n, x_n, x_{n+1}, t)\}.$$

Hence

$$\mathcal{M}(x_n, x_n, x_{n+1}, t) > \mathcal{M}(x_{n-1}, x_{n-1}, x_n, t) \text{ for all } n \geq 1$$

Thus $\{\mathcal{M}(x_n, x_n, x_{n+1}, t)\}$ is monotonically increasing sequence of positive real numbers boundedabove by 1, It is convergent to a positive real number, say L.

Therefore

$$\lim_{n\to\infty} \mathcal{M}(x_n, x_n, x_{n+1}, t) = L$$

$$\lim_{k\to\infty} \mathcal{M}(x_{n_k}, x_{n_k}, x_{n_k+1}, t) = L \quad \{\mathcal{M}(x_n, x_n, x_{n+1}, t)\} \quad \{\mathcal{M}(x_{n_k}, x_{n_k}, x_{n_k+1}, t)\}$$ Also the sequence has a sub sequence converges to L.

To prove that L = 1Suppose L < 1

$x_n = T^nx_0$ x_{n_k}, $\lim_{k\to\infty} \mathcal{M}(x_{n_k}, x_{n_k}, u, t) = 1$ Since has a subsequence converges to uWe have → (3.4.1)

Now

$$1 > L = \lim_{k\to\infty} \mathcal{M}(x_{n_k},\ x_{n_k},\ x_{n_k+1},\ t)$$

$$\geq \lim_{k\to\infty} \{\mathcal{M}(x_{n_k},\ x_{n_k},\ u,\ t/2) * \mathcal{M}(u,\ x_{n_k+1},\ x_{n_k+1},\ t/2)$$

$\lim_{k\to\infty} \mathcal{M}(x_{n_k},\ x_{n_k},\ x_{n_k+1},\ t) = 1$ = 1 * 1 using (3.4.2)Which is contradiction.

Therefore

Now we have to prove u is fixed point of T.Suppose u ≠ Tu we have

$$\mathcal{M}(u, u, Tu, t) = \lim_{k\to\infty} \mathcal{M}(x_{n_k+1}, x_{n_k+2}, Tu, t)$$

$$= \lim_{k\to\infty} \mathcal{M}(T x_{n_k}, T x_{n_k+1}, Tu, t)$$

$$> \lim_{k\to\infty} \min\{\mathcal{M}(x_{n_k}, x_{n_k+1}, u, t), \mathcal{M}(x_{n_k}, T x_{n_k}, T x_{n_k+1}, t),$$

$$\mathcal{M}(u, T x_{n_k+1}, Tu, t)\}$$

$$> \lim_{k\to\infty} \min\{\mathcal{M}(x_{n_k}, x_{n_k+1}, u, t), \mathcal{M}(x_{n_k}, x_{n_k+1}, x_{n_k+2}, t),$$

$$\mathcal{M}(u, x_{n_k+2}, Tu, t)\}$$

$$= \min\{\mathcal{M}(u, u, u, t), \mathcal{M}(u, u, u, t), \mathcal{M}(u, u, Tu, t)\}$$

$$= \mathcal{M}(u, u, Tu, t)$$

Which is contradiction to u ≠ Tu?Thus u = Tu.

Uniquness: Suppose there exists v∈ X such that Tv = v and v ≠ u.Now consider

$$\mathcal{M}(u, u, v, t) = \mathcal{M}(Tu, Tu, Tv, t)$$

$$> \min\{\mathcal{M}(u, u, v, t), \mathcal{M}(u, Tu, Tu, t), \mathcal{M}(v, Tu, Tv, t)\}$$

$$= \min\{\mathcal{M}(u, u, v, t), \mathcal{M}(u, u, u, t), \mathcal{M}(v, u, v, t)\}$$

$$> \mathcal{M}(u, u, v, t)$$

Which is contradiction?

Therefore, u is a unique fixed point of T.

$x \in X$ and $t > 0$. $\mathcal{M}(Tx, T^2x, T^3x, t) > \mathcal{M}(x, Tx, T^2x, t)$ Theorem 3.4.2 Let (X, M, *) be a generalized fuzzy metric space and T : X→X be a planning

$\{x_{n_k}\}$ with the end goal that for all for any $x_0 \in X$

$x_n = (T^n x_o)$ the sequence has a subsequence converges to u.Then u is unique fixed point of T.

Proof:

Let $x_0 \in X$ be any arbitrary fixed element in X

$x_{n+1} = Tx_n = T^{n+1}x_o$

Define the sequence for all n ≥ 1.If for some $x_{n+1} = x_n$.

Thus $Tx_n = x_n$.

Hence $u = x_n$ fixed point of T.

$x_{n+1} \neq x_n$ We assume that for all n.For n ≥ 1 we have

$$\mathcal{M}(x_n, x_n, x_{n+1}, t) = \mathcal{M}(Tx_{n-1}, Tx_{n-1}, Tx_n, t)$$

$$> \mathcal{M}(x_{n-1}, x_{n-1}, x_n, t)$$

Thus $\{\mathcal{M}(x_n, x_n, x_{n+1}, t)\}$ is monotonically increasing sequence of positive real numbers bounded aboveby 1, it is convergent a positive real number, say L ≤ 1.

Hence $\{\mathcal{M}(x_n, x_n, x_{n+1}, t)\}$ has a subsequence $\{\mathcal{M}(x_{n_k}, x_{n_k}, x_{n_k+1}, t)\}$ which is convergent to l

Also the sequence $\{\mathcal{M}(x_n, x_n, x_{n+1}, t)\}$ has a sub sequence $\{\mathcal{M}(x_{n_k}, x_{n_k}, x_{n_k+1}, t)\}$ that is converges toL

$$\lim_{k\to\infty} \mathcal{M}\,(x_{n_k}, x_{n_k}, x_{n_k+1}, t) = L$$

$\{x_{n_k}\}$ Now, we have to prove that L = 1Suppose L < 1.

Since {xn } has a subsequence converges to uWe have $\lim_{k\to\infty} \mathcal{M}\,(x_{n_k}, x_{n_k}, u, t) = 1$

Brouwer (2012) proved the fixed point theorem, which asserts that a continuous mapping of a closed unit ball in n-dimensional Euclidean space must have at least one fixed point. There are many proofs of this fundamental fact in the literature. Alexendroff and Hopf (2013) utilised algebraic topology techniques to show Brouwer's theorem, but Birkhoff and Kellogg and Dunford and Schwartz used classical analysis and determinant methods to prove the same theorem. Theorems restricted to R's subspaces are of little use in functional analysis, where the circumstance of E being an infinite dimensional subset of some function space is often encountered. Birkhoff and Kellogg were the first to prove the infinite dimensional fixed point theorem almost four decades ago.

In fact, Birkhoff and Kellogg utilised Brouwer's fixed point theorem in 1922 to prove the existence theorems in differential equations theory. The fixed point theorem of Brouwer was then expanded by Schaiider (2014) to the situation when E is a compact convex subset of anormed space.

TychonofiF (2015) went on to generalise Schaiider's conclusions from normed spaces to any locally convex space. Banach proved the fixed-point theorem for contraction maps, which is well-known for its ease of demonstration and lack of topological background.

Kannan (2016), Husain and Sehgal, Caristi, and others have looked at numerous generalisations of contraction maps in recent years and found a variety of findings.

Chu and Diaz and Bryant found that given a continuous mapping T of a full metric space into itself such that T" is a contraction mapping of X for any positive integer k, T has a unique fixed point. Rakotch and Boyd-Wong sought to expand the Banach Contraction Theorem by substituting any real valued function whose values are in the Banach Contraction Theorem (2016).

FIXED POINT:

Fixed point theory is a broad area with several applications in science and engineering. It is a key tool for topology and analysis. In analysis, fixed point is utilised to solve a variety of differential and integral problems. Theorems of fixed points are mostly utilised to solve functional equations. The fixed point of the transformation is a point that stays unchanged throughout the transformation. Fixed point theory was initially proposed in the study of vector distribution of a function by the French mathematician Poincare (1886). It was later frequently employed in mathematical analysis to answer existence difficulties. In many domains, fixed point theory is helpful for describing equilibrium. Fixed point theory is significant in differential equations, integral equations, partial differential equations, operator equations, and functional equations that occur in several fields such as finance mathematics, stability theory, economics, game theory, and dynamic programming.

Jungck investigates the presence of common fixed points in the context of commuting and compatible mappings (1976, 1986). Others who developed the idea include Popa (1990) and Pant (1999); more specifically, various scholars extended the theory in the contexts 5 of weakly commuting, R-weakly commuting, and weakly compatible mappings. Su et al. investigate the concept of multivariate fixed points in an N-variable mapping that meet certain acceptable contractive criteria established on a full metric space (2016).

Babakhani proves the existence of a solution for a linked system of fractional integro-differential equations using the Schauder fixed point theorem (2013). Using the Banach fixed point theorem, Anber et al. (2013) showed the unique existence of a solution for a class of nonlinear fractional differential equations boundary value problems with integral boundary conditions. Zada et al. (2018) examined the existence of the unique solution for fractional differential equations by defining L-cyclic (,) s contraction in b-metric spaces.

Using the idea of simulation functions, Khojasteh et al. (2015) devised a new form of contraction termed Z-contraction; Roldan Lopez-de Hierro et al. (2015) somewhat changed the notion of simulation functions and proved several fixed point theorems. Ran and Reurings (2003) developed the idea of partly ordered metric space and demonstrated a Banach counterpart.

Using a mixed monotone condition, Bhaskar and Lakshmikantham (2006) established a linked fixed point theorem in a partly ordered metric space, which was further expanded by Sintunavarat et al (2012). In the framework of partly ordered cone metric space, Sabetghadam et al. (2009) expanded the theory ofBhaskar et al. Luong et al. (2013) and Sedghi et al. (2013) add to it (2014).

Ordinary differential equations and fractional stochastic differential equations are widely recognised for their importance in the construction of compartmental epidemic models. Wang et al. (2010, 2012), Sedghi et al. (2014), Nadeem and Dabas (2016), and Kumar et al. (2016) have all published important papers that combine impulsive fractional differential equations with fixed point theory (2018).

Wardowski (2012) developed the notion of F-contraction, which was then expanded by Wardowski and Dung (2014) as F-weak contraction; Dung and Hang (2015), Hussain et al. (2015), Piri and Kumam (2016) are some of the others who explored and extended the theory further. Piri and Kumam define the Suzuki F-contraction notion (2014).

Sgroi and Vetro (2013) proposed the idea of closed multivalued F-contraction and used Hardy-Rogers type multivalued F-contractions on full metric spaces to establish several fixed-point theorems. In the settings of F-contraction and Multivalued extended F-contraction, Ahmad et al. (2015) demonstrated various fixed-point theorems utilising two new classes of control functions and a Suzuki-Hardy-Rogers type fixed point theorem.

CONCLUSION

Some of the primary elements of this research effort include the study of numerous "fixed point" outcomes in the area of FMS, such as "intuitionistic fuzzy metric spaces ()" and "- fuzzy metric spaces (-FMS)". We employ the CLRg and CLRST qualities to relax numerous constraints such as continuity, range confinement, and subspace closure, among others. In this newly defined space, we also propose the idea of "modified intuitionistic - fuzzy metric space (MI -FMS)" and investigate common and linked fixed point theorems. Our "fuzzy fixed point" findings also have some applications in the realm of "dynamic programming." There are eight chapters in this thesis, followed by references and a list of publications. In the first chapter, we provide an overview of our study issue, as well as its significance and applications. We discuss some of the domains in which "fuzzy fixed point theorems" are used. We also briefly describe the goals of our study and the organization of the various parts of the thesis. We provide fresh discoveries in several domains such as FMS, IFMS, MIFMS, and -FMS in this thesis. We also look at how our discoveries on "fixed point theorems" might be used to "dynamic programming." Furthermore, by changing the concept of -FMS, we propose the innovative notion of MI -FMS. Following that, we look at some "fixed point" findings in the newly specified MI -FMS. The range of "weakly compatible mappings" is broadened to include "OWC mappings" in some preceding "fixed point" findings. We look at how our results may be used to "dynamic programming." We propose the term "modified intuitionist -fuzzy metric space" as a new concept. We may use a single function in MI - FMS to determine the degree of nearness and non-nearness.

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References

[1] F. Riesz, B. Sz-Nagy, Functional analysis, Frederick Ungar Publishing Co., New York (1955)

[2] H.L. Royden, Real analysis, 3rd ed., Macmillan Publishing Company, New York (1988)

[3] W. Rudin, Principles of mathematical analysis, 3rd ed., McGraw-Hill Book Company, New York (1976)

[4] W. Rudin, Real and complex analysis, 3rd ed., McGraw-Hill Book Company, New York (1986)

[5] W. Rudin, Functional analysis, McGraw-Hill Book Company, New York (1973)

Coincident Point Theorem and Common Fixed-Point Theorems in M-Fuzzy Metric Spaces

Dr. Devendra Singh & Antima Jain

[6] Aamri M. and Moutawakil D. E. 2002. Some New Common Fixed Point Theorems under Strict Contractive Conditions. Journal of Mathematical Analysis and Applications. 270 (1): 181 - 188.

[7] Abbas. M., Ali. K. M. and Radenovic. S. 2010. Common Coupled Fixed-Point Theorems in Cone Metric Spaces for w-Compatible Mappings. Appl. Math. Comput. 217: 195 - 202.

[8] Ahmed. A. and Rhoades. B. E. 2001. Some Common Fixed Point Theorems for Compatible Mapping. Indian J. Pure and Appl. Math. 32: 1247-1254.

[9] Ahmed. M. A. and Nafadi. H. A. 2013. Common Fixed Point Theorems for Hybrid Pairs of Maps in Fuzzy Metric Spaces. Journal of the Egyptian Mathematical Society. Http://Dx.Doi.Org/10.1016/J.Joems. 10.006.

[10] Alaca. C., Turkoglu. D. and Yildiz. C. 2006. Fixed Points in Intuitionistic Fuzzy Metric Spaces. Chaos, Solitons & Fractals. 29: 1073 - 1078.

[11] Al - Thagafi., M. A. And Shahzad N. 2008. Generalized I -Nonexpansive Selfmaps and Invariant Approximations. Acta Math. Sinica. 24(5): 867 - 876.

[12] Ali. J., Imdad M. And Bahuguna D. 2010. Common Fixed Point Theorems in Menger Spaces with Common Property (E. A). Comput. Math. Appl. 60(12): 3152 - 3159.

[13] Bellman. R. 1957. Dynamic Programming. Princeton Univ. Press. Princiton N.J. 150 • Bellman. R. and Roosta. M. 1982. A Technique for the Reduction of Dimensionality in Dynamic Programming. J. of Math. Anal. and Appl. 88: 543 - 546.

[14] Bhakta. P. C. and Choudhury S. R. 1988. Some Existence Theorems for Functional Equations Arising in Dynamic Programming. II. J. Math. Anal. Appl. 131(1): 217 - 231.

[15] Chang S. S. and Ma Y. H. 1991. Coupled Fixed Points for Mixed Monotone Condensing Operators and An Existence Theorem of The Solutions for A Class of Functional Equations Arising in Dynamic Programming. J. Math. Anal. Appl. 160(2): 468-479.

[16] Cho. Y. J. 1997. Fixed Points in Fuzzy Metric Spaces. Journal of fuzzy mathematics, 5(4): 949- 952.