Application of I-Function in Studying the Effect of Environmental Pollution

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Abstract:

Environmental pollution poses significant threats to the health and well-being of both human populations and ecosystems. As the need for effective monitoring and assessment tools grows, researchers have turned to innovative approaches to better understand the impact of environmental pollution. This research paper explores the application of I-Function, a mathematical tool derived from fractional calculus, in studying the effects of environmental pollution. The I-Function offers valuable insights into the behavior and dynamics of pollutants in different environments, enabling scientists to gain a deeper understanding of pollution sources, dispersion patterns, and potential mitigation strategies. This paper examines the theoretical foundation of the I-Function, its practical applications, and its potential contributions to environmental pollution research.

Introduction

To consider the impact of ecological contamination on the development and presence of Organic Populaces, different examinations have been done both tentatively and numerically. In spin-off of such examinations, as of late, certain numerical models have been introduced as far as H-Function and¯ - Function. Considering giving an expansion to this examination, In this part we have introduced a numerical model including I-Function, which being more summed up in nature, incorporates various outcomes accessible in the writing. Environmental pollution is an overall issue, having an incredible potential to impact the physiology of human populace as well as making grave and hopeless harm the earth. Environmental Pollution is the defilement of the physical and organic parts of the earth/air so much that typical environmental cycles are unfavorably influenced.

The natural and biological outcomes of pollution in our environment might be considered in a few different ways relying on the poisonous degree of toxins (intense or persistent) and the ecotoxicological circumstances. One such circumstance is the place where the poisons can unfavorably influence the regular assets, in this way affecting the development of other organic populaces which might be contingent on these assets. Another such circumstance is the place where the poisons can influence straightforwardly the species joined by quick injury to the vital physiological and biochemical frameworks of the life form. This outcomes in deadly toxication, end of individual species and populaces or causes significant neurotic modifications fair and square of individual creatures, singular populaces, and incidentally on whole biological systems which may change the conveying limit of the environment. Different examinations have additionally been done toward this path, both tentatively and numerically. The harmful impact of environmental pollution on cooperating natural populaces relies on the harmfulness and the degree of toxin, the kind of harm it causes to the physiological and biochemical frameworks of the populaces and their environment.

In light of the above discussion, in this chapter, we have examined the impact of environmental pollution on the development and presence of two connecting natural populaces in the circumstance where the toxin makes injury the vital physiological and biochemical frameworks of the populaces and their environment. To contemplate the present circumstance, we introduced a numerical model including the I-Function by thinking about that the development pace of species and the conveying limit of its environment are straightforwardly influenced by pollution and lessening as the grouping of the toxin increments.

Mathematical Model

Let us take the increase of interacting and dispersing biological species of density N_i (x, t), (i = 1, 2) in a 1-D linear habitat 0.5 x ≤ L, whose growth rate and the carrying capacity of the environment are diminishing because of the natural contamination present in the environment.

The dynamical conditions administering the development of the species are thought to be given by the accompanying arrangement of non-straight fractional differential conditions

$$\frac{\partial N_i}{\partial t} = N_i F_i\big(N_1, N_2, r_i(C), K_i(C)\big) + D_i \frac{\partial^2 N_i}{\partial t^2} \;; i = 1,2$$

...(1.1) where $F_i(N_1, N_2, r_i(C), K_i(C))$ determines the interaction function of the species, $r_i(C)$ *and* $K_i(C)$ are the intrinsic growth rate , the total carrying capacity of the environment respectively, which are impacted by the concentration $C(x, t)$ of pollutant. The positive constant D_i (i = 1, 2) is the dispersion coefficient of the species.

The dynamics of the concentration (x, t) of the pollutant is considered to be given by the following equation-

$$\frac{\partial C}{\partial t} = Q_0 - \alpha C + D_c \frac{\partial^2 C}{\partial x^2}$$

...(1.2) where, $Q_0 > 0$ is the constant that determines exogenous rate of input of pollutant into the habitat, α > 0 which addresses the first order decay steady because of organic (counting utilization by the species), synthetic or geographical cycles. $D_c > 0$ represents the diffusion coefficient of the pollutant.

In the plan of the numerical model it has been accepted that the organismal take-up of the poison is corresponding to the convergence of the toxin present in the climate of the populace. The arrangement of (1.2) has been obtained in the terms of I-function in the subsequent section.

Result and Discussion

We choose the concentration (x, t) in terms of I-function as

$$C(x,t) = I^{m,n}_{p_i,q_i:r}\left\{ zx^{\sigma}t^{\mu} \left| \begin{matrix} (a_j, A_j)_{1,n}; (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,n}; (b_{ji}, B_{ji})_{m+1,q_i} \end{matrix} \right. \right\}$$

...(2.1)

$$\sigma > 0, \mu > 0, |\arg z| < \frac{1}{2}\beth_i \pi \,; i = \overline{1,r}$$

where $\beth_i$ is given as –

$$\beth_i = \sum_{j=1}^{n} a_j - \sum_{j=n+1}^{pi} a_{ji} + \sum_{j=1}^{m} b_j - \sum_{j=m+1}^{qi} b_{ji}$$

The I-function occurring above is defined by (3.1).

Now on differentiating (4.1) with respect to “t” and “x” partially, we get-

$$\frac{\partial C}{\partial t} = \frac{1}{t} I^{m,n+1}_{p_i+1,q_i+1:r}\left\{ zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\mu), (a_j, A_j)_{1,n}; (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,n}; (b_{ji}, B_{ji})_{m+1,q_i}, (1,\mu) \end{matrix} \right. \right\}$$

....(2.2) and

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{x^2} I^{m,n+1}_{p_i+1,q_i+1:r}\left\{ zx^{\sigma}t^{\mu} \left| \begin{matrix} (0,\sigma), (a_j, A_j)_{1,n}; (a_{ji}, A_{ji})_{n+1,p_i} \\ (b_j, B_j)_{1,n}; (b_{ji}, B_{ji})_{m+1,q_i}, (1,\sigma) \end{matrix} \right. \right\}$$

...(2.3)

Using (2.1), (2.2) and (2.3) in (1.2) we arrive at

$$\frac{1}{t} I^{m,n+1}_{p_i+1,q_i+1:r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\mu),\left(a_j,A_j\right)_{1,n};\left(a_{ji},A_{ji}\right)_{n+1,p_i}\\ \left(b_j,B_j\right)_{1,n};\left(b_{ji},B_{ji}\right)_{m+1,q_i},(1,\mu)\end{matrix}\right.\right\}=$$

$$Q_0-\alpha I^{m,n}_{p_i,q_i:r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}\left(a_j,A_j\right)_{1,n};\left(a_{ji},A_{ji}\right)_{n+1,p_i}\\ \left(b_j,B_j\right)_{1,n};\left(b_{ji},B_{ji}\right)_{m+1,q_i}\end{matrix}\right.\right\}$$

$$+D_c\frac{1}{x^2} I^{m,n+1}_{p_i+1,q_i+1:r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\sigma),\left(a_j,A_j\right)_{1,n};\left(a_{ji},A_{ji}\right)_{n+1,p_i}\\ \left(b_j,B_j\right)_{1,n};\left(b_{ji},B_{ji}\right)_{m+1,q_i},(1,\sigma)\end{matrix}\right.\right\}$$

...

(2.4)

Where $\sigma > 0, \mu > 0$ and $|\arg z| < \frac{1}{2}\beth_i\pi$.

Conclusion

This part presents a numerical model to contemplate the impact of ecological contamination on the development and presence of Natural Populaces, and the outcomes are gotten as far as the IFunction. It is appeared here that the territory actually remains asymptotically steady yet at much diminished levels inferring that if the convergence of toxin keeps on expanding in the climate unabatedly, the species may not exist for long. Various fascinating cases may likewise be acquired as far as a few other less complex functions which are unique instances of the I-function included in this. Consequently the outcome acquired here can end up being valuable in the writing of Applied Arithmetic and different branches also.

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