Effect of Two-electron Temperature Distributions on modulation Instability of Dust ion Acoustic Waves in Plasma

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Abstract

The modulational instability of dust ion-acoustic waves in a collision less plasma with warm adiabatic ions, cold dust and two temperature distribution of electrons is studied using the Krylov-Bogoliubov-Mitropolosky (KBM) perturbation technique. Nonlinear Schrödinger equation (NLSE) governing the slow modulation of the wave amplitude has been derived for the system. It is found that there exist two critical values of wave number k_{c1} and k_{c2}; which decide the modulation instability of dust ion acoustic waves. The waves are found to be modulationally unstable for the values of k lying in the range ($0 < k < k_{c1}$) and for $k > k_{c2}$.

It is also found that due to the presence of dust the dust ion-acoustic wave becomes unstable for those values of wave numbers for which it was perfectly stable in the absence of dust. The instability region increases with increase in ion temperature ratio as well as with change in polarity of dust particle from negative to positive. The region of instability also increases with an increase in cold electron concentration and with an increase in the temperature ratio of the two species of electrons. It is found that there is shift in the instability region from higher wave number to lower wave number on changing the polarity of dust grain from positive to negative. The roles of the other plasma parameters (e. g., ion temperature, mass ratio, charge multiplicity) are also discussed in detail.

Introduction

During last two decades, there has been a great deal of interest in the experimental and theoretical study of linear and nonlinear wave phenomena in dusty plasma due to omnipresent dust particles in laboratory (Homan et al. 1997), astrophysical environments and space plasma (Mendis and Rosenberg 1992; Shukla and Mamun 2003). Dust grains having masses in the range from 10^5 to 10^{12} times of the mass of the proton, with size of the order of micrometer, having charges of both polarities (Rosenberg and Mendis 1995; Rosenberg et al. 1999; Smith et al. 2001) with several order of magnitude of that of electrons/ions are found in laboratory as well as in space, and astrophysical environments e.g., in the Earth's ionosphere, Magnetosphere, planetary rings, comets, interstellar molecular clouds and circumstellar disks (Rosenberg 1993; Horanyi 1996; Boufendi et al 1999). Dusty plasma consisting of massive solid dust grains drastically change the properties of the plasma and give rise to new types of wave modes (Rao et al. 1990; Chu et al. 1994). The existence of an extremely low–phase velocity and low frequency dust acoustic wave has been studied by several authors (Varma et al. 1993; Yaroshenko et al. 2004; Zahran et al. 2013) and has also been experimentally observed (Thompson et al. 1995; Prabhakara and Tanna 1996). The dust-ion-acoustic waves (DIAW), the usual ion- acoustic mode modified by the presence of dust particles in electron-ion plasma was first pointed out by Shukla and Silin (1992). The DIAW has been theoretically investigated by several authors (Shukla and Silin 1992; Merlino et al. 1998; Mamun and Shukla 2002; Roy et al. 2007; Dubinov and Sazonkin 2013). The DIAW has also been observed experimentally in laboratory plasmas (Barkan et al. 1996; Merlino et al. 1998).

Plasmas with two-electron temperature distributions having hot and cold population are commonly observed in laboratory tokamak experiments, in thermonuclear fusion (Morales and Lee 1974; Jones et al. 1975; Lipschultz et al. 1986), in the tandem mirror device. The electron distribution composed of two Maxwellians at different temperatures is observed in the open–end region in front of end plates (Kurihara et al. 1989), in hot cathode discharge plasma (Jones et al. 1975), as well as in space by the satellites. The observations made by Various spacecraft e.g. the Fast Auroral Snapshot (FAST) at the auroral region (Ergun et al. 1998), S3-3 Satellite, THEMIS mission (Temerin et al. 1982), Viking satellite (*Boström* et al. 1988), and earlier missions GEOTAIL and POLAR (Mcfadden et al. 2003) in the magnetosphere have reported the coexistence of such electron populations. Thus it is clear that two-electron Maxwellian distribution is very common in space and laboratory experiments.

There has been a great zeal to understand nonlinear ion acoustic solitary waves, double layers and modulational instability in two-electron temperature plasmas. A large number of authors few to cite (Yadav and Sharma 1990; Mishra et al. 2007; Bharuthram et al. 2008; Jain and Mishra 2013) have studied nonlinear waves in two electron temperature plasmas. Modulational instability of obliquely modulated ion acoustic waves in collisionless plasma with two-electron temperature was investigated by Yashvir (1985) and in collisional plasma by Mishra et al. (1989). Recently nonlinear modulation of ion-acoustic waves in two electron-temperature plasmas has been studied by Esfandyari- Kalejahi et al. (2010).

Importance of modulational instability in stable wave propagation has motivated the researchers to investigate modulational instability of different wave modes in plasmas having different components (Amin et al. 1998; Kouraki and Shukla 2004; Bains et al. 2013; Chawla et al. 2013). Two-electron temperature plasma have been observed in different laboratory and space environment such as auroral and magnetospheric region where the omnipresent dust particles are also present. Therefore it becomes instructive to investigate the modulational instability in multicomponent collisionless plasma composed of warm adiabatic ions, cold mobile dust and two species of electrons.

Using the Krylov-Bogoliubov-Mitropolosky (KBM) perturbation technique a nonlinear Schrödinger equation governing the slow modulation of the wave amplitude is derived for the system. It is found that there exist two regime of instability for the dust ion acoustic waves, one below k_{c1} and other above k_{c2}. The waves are found to be modulationally unstable for the values of k lying in the range

($0 < k < k_{c1}$) and for $k > k_{c2}$. It is also investigated that the dust ions acoustic waves are unstable for lower values of wave numbers,

which was perfectly stable in the absence of dust. The region of instability increases with increase in ion temperature ratio as well as with change in polarity of dust particle from negative to positive and with an increase in cold electron concentration. The region of instability also increases with an increase in the temperature ratio of the two species of electrons.

The effect on the instability region of various plasma parameters has also been investigated. The variation in maximum growth rate and modulational wave number for different plasma parameters has also been evaluated numerically to find the region of physical instability. The results obtained in this study may be useful to explain the modulational instability of dust ion acoustic wave in the laboratory plasmas and astrophysical environments such as auroral plasma, where unmagnetized warm ions, two species of electrons and cold dust particles are present.

The paper is organized as follows: The basic set of equations is given in Section 2. The nonlinear Schrödinger equation has been derived in Sec.3. In Sec. 4 stability analysis has been discussed. The conclusions are summarized in Sec. 5.

Basic equations

We consider collisionless unmagnetized dusty plasma consisting of warm adiabatic ions having temperature T_i, cold dust and two species of isothermal electrons separately in thermal equilibrium having a hot component with density n_h and temperature T_h, and a cold component with density n_c and temperature T_c. We further assume that ion temperature is small as compared to the temperatures of electrons i.e., ($T_i \ll T_h$, T_c). We neglect the effect of particle trapping and inertia of the electrons. We have used the two fluid model to describe the behavior of the dust ion acoustic waves. The normalized basic set of equations is:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i v_i\right) = 0 \qquad (1)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = \frac{E}{\gamma} - \frac{\sigma}{\gamma} n_i \frac{\partial n_i}{\partial x} \quad (2)$$

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \quad (3)$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = \frac{M_d \varepsilon_z}{\gamma} E \quad (4)$$

$$\frac{\partial n_h}{\partial x} = -\frac{\beta}{(\mu + \nu\beta)} n_h E \quad (5)$$

$$\frac{\partial n_c}{\partial x} = -\frac{1}{(\mu + \nu\beta)} n_c E \quad (6)$$

$$\frac{\partial E}{\partial x} = (1-\alpha) n_i + \alpha n_d - n_h - n_c \quad (7)$$

In the above equations, n_i, n_d and v_i, v_d are the normalized densities and fluid velocities of the ion and dust species respectively, n_h and n_c are the electron densities for the hot and cold species, E is the electric field, T_i, T_h and T_c are the ion temperature, temperature of the hot and cold species of electrons respectively. Here $\sigma = 3T_i / T_{eff}$ defines the temperature ratio of adiabatic warm ions to the effective temperature of electrons, $\gamma = (1-\alpha + M_d \alpha)$, $M_d = (m_i / m_d)$ is the ratio of mass of ion to the mass of dust particle $\varepsilon_z = (Z_d / Z_i)$ is the ratio of charge multiplicity, $\alpha = n_{d0} / n_{e0}$, $\mu = n_{c0} / n_{e0}, \nu = n_{h0} / n_{e0}$ and $\beta = T_c / T_h$. Here n_{d0}, n_{c0}, n_{h0} and n_{e0} are the equilibrium densities of dust, two species of electrons and of the electrons respectively.

The densities n_i, n_d are normalized with respect to their corresponding equilibrium densities n_{i0} and n_{d0}. The velocity with respect to the effective ion acoustic speed $C_{eff} = (T_{eff} / m_i)^{\frac{1}{2}}$, with $T_{eff} = n_0 T_h T_c (n_{c0} T_h + n_{h0} T_c)^{-1}$, the length with respect to the effective Debye length $\lambda_{Deff} = (T_{eff} / 4\pi n_{e0} e^2)^{\frac{1}{2}}$ and the time t with inverse of ion plasma frequency ω_{pi}^{-1}. The electric field E is normalized with $T_{eff} / e\lambda_{Deff}$ .The charge neutrality condition is expressed as $(n_{i0} / n_{e0}) = (\mu + \nu) - Z_d (n_{d0} / n_{e0})$.

Using the KBM perturbation method for nonlinear wave modulation, we expend all the quantities about their equilibrium state as follows:

$$E = \varepsilon E_1 + \varepsilon^2 E_2 + \varepsilon^3 E_3 + \ldots\ldots\ldots\ldots\ldots\ldots$$
$$v_i = \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \varepsilon^3 v_{i3} + \ldots\ldots\ldots\ldots\ldots\ldots$$
$$v_d = \varepsilon v_{d1} + \varepsilon^2 v_{d2} + \varepsilon^3 v_{d3} + \ldots\ldots\ldots\ldots\ldots\ldots$$
$$n_i = 1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \ldots\ldots\ldots\ldots\ldots$$
$$n_d = 1 + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \varepsilon^3 n_{d3} + \ldots\ldots\ldots\ldots$$
$$n_h = \nu + \varepsilon n_{h1} + \varepsilon^2 n_{h2} + \varepsilon^3 n_{h3} + \ldots\ldots\ldots\ldots$$
$$n_c = \mu + \varepsilon n_{c1} + \varepsilon^2 n_{c2} + \varepsilon^3 n_{c3} + \ldots\ldots\ldots\ldots \quad (8)$$

To study the modulational instability of dust ion-acoustic waves in the system, we assume that the perturbed quantities of all orders depend on x and t through the complex amplitudes (a , $\overline{a}$) and the phase factor (ψ). The phase factor is given by $\psi = kx - \omega t$. The wave vector and frequency are k and ω respectively for the dust ion-acoustic waves.

The complex amplitude a is a slowly varying function of x and t expressed as

$$\frac{\partial a}{\partial t} = \varepsilon A_1(a,\overline{a}) + \varepsilon^2 A_2(a,\overline{a}) + \varepsilon^3 A_3(a,\overline{a}) + \ldots\ldots\ldots\ldots\ldots \quad (9a)$$

$$\frac{\partial a}{\partial x} = \varepsilon B_1(a,\overline{a}) + \varepsilon^2 B_2(a,\overline{a}) + \varepsilon^3 B_3(a,\overline{a}) + \ldots\ldots\ldots\ldots\ldots\ldots \quad (9b)$$

We employed Eq. (9) along with complex conjugate relations to determine the unknown functions A_1, A_2,...and B_1, B_2,... by eliminating all secular terms in the perturbation solution.

Derivation of the nonlinear Schrödinger equation (NLSE)

In the basic set of equations (1)-(7), using Eq. (8) and (9) we obtained a set of equations for each order in ε , on equating terms with the same power of ε. The first order solutions are as follows:

$$n_{i1} = a\exp(i\psi) + \overline{a}\exp(-i\psi)$$

$$E_1 = \frac{i}{k}\left[\left(\sigma k^2 - \omega^2\right)\left\{a\exp(i\psi) - \overline{a}\exp\left(-i\psi\right)\right\}\right]$$

$$v_{i1} = \frac{\omega}{k}\left[a\exp(i\psi) + \bar{a}\exp(-i\psi)\right]$$

$$n_{d1} = \frac{M_d \varepsilon_z}{\gamma\omega^2}\left(\gamma\omega^2 - \sigma k^2\right)\left[a\exp(i\psi) + \bar{a}\exp(-i\psi)\right]$$

$$v_{d1} = \frac{M_d \varepsilon_z}{\gamma\omega k}\left(\gamma\omega^2 - \sigma k^2\right)\left[a\exp(i\psi) + \bar{a}\exp(-i\psi)\right]$$

$$n_{h1} = \frac{\nu\beta}{k^2(\mu+\nu\beta)}\left[\left(\gamma\omega^2 - \sigma k^2\right)\left\{a\exp(i\psi) + \bar{a}\exp(i\psi)\right\}\right]$$

$$n_{c1} = \frac{\mu}{k^2(\mu+\nu\beta)}\left[\left(\gamma\omega^2 - \sigma k^2\right)\left\{a\exp(i\psi) + \bar{a}\exp(i\psi)\right\}\right] \quad (10)$$

The first- order equations give rise the dispersion relation for the dust ion-acoustic waves as

$$1 + \frac{1}{k^2} = \frac{M_d \alpha \varepsilon_z}{\gamma\omega^2} + \frac{(1-\alpha)}{\left(\gamma\omega^2 - \sigma k^2\right)} \quad (11)$$

The above equation reduces to Yashvir et al.[35] in case of absence of dust and cold ions i. e., considering $n_d = 0,\ \gamma = 1,\ \alpha = 0, and\, \sigma = 0.$

The set of second order equations can be written as:

$$\omega\frac{\partial n_{i2}}{\partial\psi} - A_1\frac{\partial n_{i1}}{\partial a} - B_1\frac{\partial v_{i1}}{\partial a} - k\frac{\partial}{\partial\psi}\left(n_{i1}v_{i1} + v_{i2}\right) = 0 \quad (12a)$$

$$\omega\frac{\partial v_{i2}}{\partial\psi} - A_1\frac{\partial v_{i1}}{\partial a} - kv_{i1}\frac{\partial v_{i1}}{\partial\psi} - \frac{\sigma}{\gamma}B_1\frac{\partial n_{i1}}{\partial a} - \frac{\sigma k}{\gamma}\frac{\partial n_{i2}}{\partial\psi} - \frac{\sigma k}{\gamma}n_{i1}\frac{\partial n_{i1}}{\partial\psi} + \frac{E_2}{\gamma} = 0 \quad (12b)$$

$$\omega\frac{\partial n_{d2}}{\partial\psi} - A_1\frac{\partial n_{d1}}{\partial a} - B_1\frac{\partial v_{d1}}{\partial a} - k\frac{\partial}{\partial\psi}\left(n_{d1}v_{d1} + v_{d2}\right) = 0 \quad (12c)$$

$$\omega\frac{\partial v_{d2}}{\partial\psi} - A_1\frac{\partial v_{d1}}{\partial a} - kv_{d1}\frac{\partial v_{d1}}{\partial\psi} + \frac{M_d\varepsilon_z}{\gamma}E_2 = 0 \quad (12d)$$

$$k\frac{\partial n_{h2}}{\partial\psi} + B_1\frac{\partial n_{h1}}{\partial a} + \frac{\beta}{(\mu+\nu\beta)}\left(n_{h1}E_1 + \nu E_2\right) = 0 \quad (12e)$$

$$k\frac{\partial n_{c2}}{\partial \psi}+B_1\frac{\partial n_{c1}}{\partial a}+\frac{1}{(\mu+\nu\beta)}(\mu E_2+n_{c1}E_1)=0 \tag{12f}$$

$$k\frac{\partial E_2}{\partial \psi}+B_1\frac{\partial E_1}{\partial a}-(1-\alpha)n_{i2}-\alpha n_{d2}-(n_{h2}+n_{c2})=0 \tag{12g}$$

Using first–order solutions and the set of second order equations, we find that second order equation in v_2 as

$$\frac{k}{\omega}\left(\gamma\omega^2-\sigma k^2\right)\left[\frac{\partial^3 v_2}{\partial\psi^3}+\frac{\partial v_2}{\partial\psi}\right]=iAa^2\exp(2i\psi)+\left[A_1+v_gB_1\right]\exp(i\psi)+c.c. \tag{13}$$

Where

$$A=\begin{bmatrix}\frac{1}{k^4}\frac{\left(\mu+\nu\beta^2\right)}{\left(\mu+\nu\beta\right)^2}\left(\gamma\omega^2-\sigma k^2\right)^2-\frac{3M_d^2\alpha\varepsilon_z^2}{\gamma^2\omega^4}\left(\gamma\omega^2-\sigma k^2\right)^2-2(1-\alpha)\\ -4(\gamma\omega^2+3\sigma k^2)+\left(\frac{M_d\alpha\varepsilon_z k^2-\gamma\omega^2}{k\gamma\omega^2}\right)\left(\frac{\gamma\omega^2+3\sigma k^2}{k}\right)\end{bmatrix} \tag{14}$$

and v_g, represents the group velocity, given by

$$v_g=\frac{d\omega}{dk}=\frac{\omega}{k}\left[\frac{M_d\alpha\varepsilon_z\left(3\sigma k^2-\gamma\omega^2\right)+\gamma\omega^2\left\{\sigma\left(3k^2+1\right)+(1-\alpha)\right\}+\gamma^2\omega^4\left(1-\frac{1}{k^2}\right)}{2M_d\alpha\varepsilon_z\left(\sigma k^2-\gamma\omega^2\right)+\frac{2\gamma^2\omega^4(1-\alpha)}{\left(\sigma k^2-\gamma\omega^2\right)}}\right] \tag{15}$$

We see that the solution of equation (13) would contain a secular term which is proportional to ψ unless the coefficient of $\exp(\pm i\psi)$ term vanishes. For second-order solutions to be non secular this gives a condition:

$$A_1+v_gB_1=0 \tag{16}$$

In the lowest order of ε, using equations (9a) – (9b), A_1 and B_1 can be regarded as $\frac{\partial a}{\partial t_1}$ and $\frac{\partial a}{\partial x_1}$ where $t_1=\varepsilon t$ and $x_1=\varepsilon t$. Thus equation (16) can be interpreted as

$$\frac{\partial a}{\partial t_1} + v_g \frac{\partial a}{\partial x_1} = 0 \qquad (17)$$

Which shows that, to the lowest order in ε, amplitude a is constant in a frame of reference moving with the group velocity. Under the condition (16), the non-secular solution of (13) is given by

$$v_{i2} = \alpha_1 a^2 \exp(2i\psi) + b_1(a,\bar{a})\exp(i\psi) + c.c. + \gamma_1(a,\bar{a}) \qquad (18\text{ a})$$

where $b_1(a,\bar{a})$ is assumed to be complex and $\gamma_1(a,\bar{a})$ is assumed to be real and both are independent of ψ but depend on a and $\bar{a}$. They are to be determined from the condition that higher order solutions should be free from secularity. Here α_1 is given by the relation $\alpha_1 = \dfrac{\omega A}{6k(\gamma\omega^2 - \sigma k^2)}$.

Using the set of second order equations (12), the remaining second order solutions can be expressed as:

$$E_2 = \alpha_2 a^2 \exp(2i\psi) + b_2(a,\bar{a})\exp(i\psi) + c.c. + \gamma_2(a,\bar{a}) \qquad (18\text{ b})$$

$$n_{i2} = \alpha_3 a^2 \exp(2i\psi) + b_3(a,\bar{a})\exp(i\psi) + c.c. + \gamma_3(a,\bar{a}) \qquad (18\text{ c})$$

$$n_{d2} = \alpha_4 a^2 \exp(2i\psi) + b_4(a,\bar{a})\exp(i\psi) + c.c. + \gamma_4(a,\bar{a}) \qquad (18\text{ d})$$

$$n_{h2} = \alpha_5 a^2 \exp(2i\psi) + b_5(a,\bar{a})\exp(i\psi) + c.c. + \gamma_5(a,\bar{a}) \qquad (18\text{ e})$$

$$n_{c2} = \alpha_6 a^2 \exp(2i\psi) + b_6(a,\bar{a})\exp(i\psi) + c.c. + \gamma_6(a,\bar{a}) \qquad (18\text{ f})$$

$$v_{d2} = \alpha_7 a^2 \exp(2i\psi) + b_7(a,\bar{a})\exp(i\psi) + c.c. + \gamma_7(a,\bar{a}) \qquad (18\text{ g})$$

where

$$\alpha_2 = i\left[2\alpha_1 \frac{(\sigma k^2 - \gamma\omega^2)}{\omega} + 2\sigma k + \frac{(\sigma k^2 + \gamma\omega^2)}{k}\right]$$

$$\alpha_3 = \frac{(\alpha_1 k + \omega)}{\omega}$$

$$\alpha_4 = \left[\frac{3}{2}\frac{M_d^2\varepsilon_z^2}{\gamma^2\omega^4}(\gamma\omega^2 - \sigma k^2)^2 - \frac{1}{6}\frac{M_d\varepsilon_z}{\gamma\omega^2}(3\gamma\omega^2 + 9\sigma k^2 - A)\right]$$

$$\alpha_5 = \left[-\frac{1}{6}\frac{\nu\beta}{(\mu+\nu\beta)}\frac{(3\gamma\omega^2+9\sigma k^2-A)}{k^2} + \frac{1}{2}\frac{\nu\beta^2}{(\mu+\nu\beta)^2}\frac{(\sigma k^2-\gamma\omega^2)^2}{k^4} \right]$$

$$\alpha_6 = \left[-\frac{1}{6}\frac{\mu}{(\mu+\nu\beta)}\frac{(3\gamma\omega^2+9\sigma k^2-A)}{k^2} + \frac{1}{2}\frac{\mu}{(\mu+\nu\beta)^2}\frac{(\sigma k^2-\gamma\omega^2)^2}{k^4} \right]$$

$$\alpha_7 = \left[-\frac{1}{6}\frac{M_d\varepsilon_z}{\gamma k\omega}(3\gamma\omega^2+9\sigma k^2-A) + \frac{1}{2}\frac{M_d^2\varepsilon_z^2}{k\gamma^2\omega^3}(\sigma k^2-\gamma\omega^2)^2 \right]$$

$$b_2 = \left[i\frac{(\sigma k^2-\gamma\omega^2)}{\omega}b_1(a,\overline{a}) + \frac{(\sigma k^2+\gamma\omega^2)}{k\omega}A_1 + 2\sigma B_1 \right]$$

$$b_3 = \left[\frac{k}{\omega}b_1(a,\overline{a}) - i\frac{A_1}{\omega} - i\frac{B_1}{k} \right]$$

$$b_4 = i\left[\frac{2M_d\varepsilon_z}{\gamma\omega^3}(\sigma k^2-\gamma\omega^2)A_1 + \frac{M_d\varepsilon_z}{\gamma k\omega^2}(\sigma k^2-\gamma\omega^2)B_1 + \frac{M_d k\varepsilon_z}{\gamma\omega^2}b_2(a,\overline{a}) \right]$$

$$b_5 = i\left[\frac{\nu\beta}{k(\mu+\nu\beta)}b_2(a,\overline{a}) + \frac{\nu\beta}{(\mu+\nu\beta)}\frac{1}{k^3}(\gamma\omega^2-\sigma k^2)B_1 \right]$$

$$b_6 = i\left[\frac{\mu}{k(\mu+\nu\beta)}b_2(a,\overline{a}) + \frac{\mu}{(\mu+\nu\beta)}\frac{1}{k^3}(\gamma\omega^2-\sigma k^2)B_1 \right]$$

$$b_7 = \left[\frac{2iM_d\varepsilon_z\sigma k}{\gamma\omega^2}A_1 + \frac{2iM_d\varepsilon_z\sigma}{\gamma\omega}B_1 + \frac{M_d\varepsilon_z}{\gamma\omega^2}(\gamma\omega^2-\sigma k^2)b_1(a,\overline{a}) \right]$$

here γ_2, γ_3, γ_4 and γ_5 are function of a and $\overline{a}$ only and are assumed to real. From equation (12b), we find that there should be no constant term in the expression for E_2. Therefore,

$$\gamma_2(a,\overline{a}) = 0 \tag{19}$$

From equations (12e) we obtain another relation

$$\gamma_5 + \gamma_6 = (1-\alpha)\gamma_3 + \alpha\gamma_4 \tag{20}$$

The set of third order equations can be written as:

$$\frac{\partial n_{i3}}{\partial \psi}=\frac{1}{\omega}\left[A_1\frac{\partial n_{i2}}{\partial a}+A_2\frac{\partial n_{i1}}{\partial a}+B_1\frac{\partial(n_{i1}v_{i1}+v_{i2})}{\partial a}+B_2\frac{\partial v_{i1}}{\partial a}+k\frac{\partial v_{i3}}{\partial \psi}+k\frac{\partial(n_{i1}v_{i2}+n_{i2}v_{i1})}{\partial \psi}\right] \quad (21\text{ a})$$

$$\frac{\partial v_{i3}}{\partial \psi}=\frac{1}{\omega}\left[\begin{array}{l}A_1\frac{\partial v_{i2}}{\partial a}+A_2\frac{\partial v_{i1}}{\partial a}+B_1v_{i1}\frac{\partial v_{i1}}{\partial a}+k\frac{\partial(v_{i1}v_{i2})}{\partial \psi}\\ +\frac{\sigma}{\gamma}\left\{B_1\frac{\partial n_{i2}}{\partial a}+B_2\frac{\partial n_{i1}}{\partial a}+B_1n_{i1}\frac{\partial n_{i1}}{\partial a}+k\frac{\partial(n_{i1}n_{i2}+n_{i3})}{\partial \psi}\right\}-\frac{E_3}{\gamma}\end{array}\right] \quad (21\text{ b})$$

$$\frac{\partial n_{d3}}{\partial \psi}=\frac{1}{\omega}\left[A_1\frac{\partial n_{d2}}{\partial a}+A_2\frac{\partial n_{d1}}{\partial a}+B_1\frac{\partial(n_{d1}v_{d1}+v_{d2})}{\partial a}+B_2\frac{\partial v_{d1}}{\partial a}+k\frac{\partial v_{d3}}{\partial \psi}+k\frac{\partial(n_{d1}v_{d2}+n_{d2}v_{d1})}{\partial \psi}\right] \quad (21\text{ c})$$

$$\frac{\partial v_{d3}}{\partial \psi}=\frac{1}{\omega}\left[A_1\frac{\partial v_{d2}}{\partial a}+A_2\frac{\partial v_{d1}}{\partial a}+B_1v_{d1}\frac{\partial v_{d1}}{\partial a}+k\frac{\partial(v_{d1}v_{d2})}{\partial \psi}-\frac{M_d\varepsilon_z}{\gamma}E_3\right] \quad (21\text{ d})$$

$$\frac{\partial n_{h3}}{\partial \psi}=-\frac{1}{k}\left[B_1\frac{\partial n_{h2}}{\partial a}+B_2\frac{\partial n_{h1}}{\partial a}+\frac{\beta}{(\mu+\nu\beta)}(\nu E_3+n_{h1}E_2+n_{h2}E_1)\right] \quad (21\text{ e})$$

$$\frac{\partial n_{c3}}{\partial \psi}=-\frac{1}{k}\left[B_1\frac{\partial n_{c2}}{\partial a}+B_2\frac{\partial n_{c1}}{\partial a}+\frac{1}{(\mu+\nu\beta)}(\mu E_3+n_{c1}E_2+n_{c2}E_1)\right] \quad (21\text{ f})$$

$$k\frac{\partial E_3}{\partial \psi}+B_1\frac{\partial E_2}{\partial a}+B_2\frac{\partial E_1}{\partial a}-(1-\alpha)n_{i3}-\alpha n_{d3}+(n_{h3}+n_{c3})=0 \quad (21\text{ g})$$

The above equations can be used to obtain an equation for v_3 similar to the equation (13) for v_2. We find that in the solution of v_3 there are resonant terms (proportional to $\exp(\pm i)$) as well as constant terms with respect to ψ, which give rise to secular behavior. Therefore, in addition to the resonant terms, we must also require that the constant terms vanish. From the later conditions together with equation (20) we can determine the unknown constants γ_1, γ_3, γ_4 and γ_5. these are

$$\gamma_1=v_g\gamma_3-2\frac{\omega}{k}a\bar{a}+c_1 \quad (22\text{ a})$$

$$\gamma_3 = \frac{\left[\left(\frac{\gamma\omega^2 + \sigma k^2 + 2v_g\gamma\omega k}{k^2}\right)\left(1 - \frac{\gamma v_g^2}{M_d\alpha\varepsilon_z}\right) + \frac{M_d\varepsilon_z(2v_g k - \omega)}{\gamma k^2\omega^3}(\gamma\omega^2 - \sigma k^2)^2 + \frac{\gamma v_g^2}{M_d\alpha\varepsilon_z k^4}\frac{(\mu + \nu\beta^2)}{(\mu + \nu\beta)^2}(\gamma\omega^2 - \sigma k^2)^2\right]}{\left[\gamma v_g^2 + \sigma - \frac{\gamma v_g^2}{M_d\alpha\varepsilon_z}(\alpha + \sigma + \gamma v_g^2 - 1)\right]} a\bar{a} + c_3 \tag{22 b}$$

$$\gamma_4 = \frac{(\alpha + \sigma + \gamma v_g^2 - 1)}{\alpha}\gamma_3 + \frac{1}{\alpha}\left[\frac{(\mu + \nu\beta^2)}{(\mu + \nu\beta)^2 k^4}(\sigma k^2 - \gamma\omega^2)^2 - \frac{\gamma\omega^2}{k^2} - \sigma - 2v_g\gamma\frac{\omega}{k}\right]a\bar{a} + c_4 \tag{22 c}$$

$$(\gamma_5 + \gamma_6) = (1 - \alpha)\gamma_3 + \alpha\gamma_4 + c_5 \tag{22 d}$$

$$\gamma_7 = v_g\gamma_4 + \frac{2M_d^2\varepsilon_z^2}{k\gamma^2\omega^3}(\gamma\omega^2 - \sigma k^2)^2 a\bar{a} + c_7 \tag{22 e}$$

where c_1, c_3, c_4, c_5 and c_7 are arbitrary constants independents of a, $\bar{a}$ and ψ can be determined by the initial conditions.

In order that the solution v_3 be nonsecular, we equate the coefficients of the resonant terms to zero. Thus we obtain

$$i(A_2 + v_g B_2) + P\left(B_1\frac{\partial B_1}{\partial a} + \bar{B}_1\frac{\partial B_1}{\partial \bar{a}}\right) + Q|a|^2 a + Ra = 0 \tag{23}$$

Since $A_2 = \frac{1}{\varepsilon^2}\frac{\partial a}{\partial t} - \frac{1}{\varepsilon}A_1,\quad B_2 = \frac{1}{\varepsilon^2}\frac{\partial a}{\partial x} - \frac{1}{\varepsilon}B_1,$

$$B_1\frac{\partial B_1}{\partial a} + \bar{B}_1\frac{\partial B_1}{\partial \bar{a}} = \frac{1}{\varepsilon^2}\frac{\partial^2 a}{\partial x^2}$$

Eqn. (23) can be written as

$$i\frac{1}{\varepsilon^2}\left(\frac{\partial a}{\partial t}+v_g\frac{\partial a}{\partial x}\right)+\frac{1}{\varepsilon^2}P\frac{\partial^2 a}{\partial x^2}+Q|a|^2 a+Ra=0 \tag{24}$$

Now using the coordinate transformation, defined by

$$\xi=\varepsilon(x-v_g t), \qquad \tau=\varepsilon^2 t \tag{25}$$

Equations (24) reduces to

$$i\frac{\partial a}{\partial \tau}+P\frac{\partial^2 a}{\partial \xi^2}+Q|a|^2 a+Ra=0 \tag{26}$$

In the above equation the linear interaction term R is not important because R is real and this causes simply a phase shift. Using a simple substitution

$$a\rightarrow a\exp(iR\tau) \tag{27}$$

Equation (26) reduces to

$$i\frac{\partial a}{\partial \tau}+P\frac{\partial^2 a}{\partial \xi^2}+Q|a|^2 a=0 \tag{28}$$

where

$$P=\frac{1}{2}\frac{dv_g}{dk}$$

$$P=-\left[\begin{array}{l}\left\{\frac{(3\sigma k^2+\gamma\omega^2)}{k^5}-\frac{3\sigma}{k}\frac{(1-\alpha)}{(\gamma\omega^2-\sigma k^2)}\right\}+v_g\left\{\frac{(3\sigma k^2-\gamma\omega^2)}{\gamma k^3\omega}(\gamma+\gamma k^2+M_d\varepsilon_z k)-\frac{(1-\alpha)}{(\gamma\omega^2-\sigma k^2)}\frac{(5\sigma k^2-\gamma\omega^2)}{k^2\omega}\right\}\\ +v_g^2\left\{\frac{(3\sigma k^2-\gamma\omega^2)}{\gamma k^2\omega^2}M_d\varepsilon_z+\frac{2\sigma k}{\omega^2}\left(1+\frac{1}{k^2}-\frac{(1-\alpha)}{(\gamma\omega^2-\sigma k^2)}\right)+\frac{\gamma(1-\alpha)}{k(\sigma k^2-\gamma\omega^2)}\right\}\end{array}\right]$$

$$\times\left[\frac{2M_d\alpha\varepsilon_z(\sigma k^2-\gamma\omega^2)}{\gamma\omega^3}+\frac{2\gamma\omega(1-\alpha)}{(\sigma k^2-\gamma\omega^2)}\right]^{-1} \tag{29}$$

and $$Q=-\left[\begin{array}{l}\left(\begin{array}{l}\left\{\frac{2\gamma\omega(1-\alpha)}{\left(\gamma\omega^2-\sigma k^2\right)}\right\}\left\{\alpha_1+\gamma_1'\right\}-\left\{i\frac{\left(\gamma\omega^2-\sigma k^2\right)}{k^4}\frac{\left(\mu+\nu\beta^2\right)}{(\mu+\nu\beta)^2}\right\}\left\{\alpha_2+\gamma_2'\right\}\\+\left\{\frac{(1-\alpha)}{k}\frac{\left(\sigma k^2+\gamma\omega^2\right)}{\left(\gamma\omega^2-\sigma k^2\right)}\right\}\left\{\alpha_3+\gamma_3'\right\}+\left\{\frac{M_d\alpha\varepsilon_z}{\gamma k\omega^2}\left(\gamma\omega^2-\sigma k^2\right)\right\}\left\{\alpha_4+\gamma_4'\right\}\\+\left\{\frac{1}{k^3}\frac{\left(\gamma\omega^2-\sigma k^2\right)}{(\mu+\nu\beta)}\right\}\left\{\alpha_5+\alpha_6-\left(\gamma_5'+\gamma_6'\right)\right\}+\left\{\frac{M_d\alpha\varepsilon_z\left(k^2+1\right)}{\gamma\omega^3}\left(\gamma\omega^2-\sigma k^2\right)\right\}\left\{\alpha_7+\gamma_7'\right\}\end{array}\right)\\\times\left\{\frac{2M_d\alpha\varepsilon_z\left(\sigma k^2-\gamma\omega^2\right)}{\gamma\omega^3}+\frac{2\gamma\omega(1-\alpha)}{\left(\sigma k^2-\gamma\omega^2\right)}\right\}^{-1}\end{array}\right] \tag{30}$$

$$R=-\left[\begin{array}{l}\left(\begin{array}{l}\left\{\frac{2\gamma\omega(1-\alpha)}{\left(\gamma\omega^2-\sigma k^2\right)}\right\}\times c_1-\left\{i\frac{\left(\gamma\omega^2-\sigma k^2\right)}{k^4}\frac{\left(\mu+\nu\beta^2\right)}{(\mu+\nu\beta)^2}\right\}\times c_2\\+\left\{\frac{(1-\alpha)}{k}\frac{\left(\sigma k^2+\gamma\omega^2\right)}{\left(\gamma\omega^2-\sigma k^2\right)}\right\}\times c_3+\left\{\frac{M_d\alpha\varepsilon_z}{\gamma k\omega^2}\left(\gamma\omega^2-\sigma k^2\right)\right\}\times c_4\\+\left\{\frac{1}{k^3}\frac{\left(\gamma\omega^2-\sigma k^2\right)}{(\mu+\nu\beta)}\right\}\times c_5+\left\{\frac{M_d\alpha\varepsilon_z\left(k^2+1\right)}{\gamma\omega^3}\left(\gamma\omega^2-\sigma k^2\right)\right\}\times c_7\end{array}\right)\\\times\left\{\frac{2M_d\alpha\varepsilon_z\left(\sigma k^2-\gamma\omega^2\right)}{\gamma\omega^3}+\frac{2\gamma\omega(1-\alpha)}{\left(\sigma k^2-\gamma\omega^2\right)}\right\}^{-1}\end{array}\right] \tag{31}$$

The coefficient P and Q depend on the parameters of the plasma system as given by Eqs. (29) and (30) respectively. Note, that for simplicity, we have dropped the linear interaction term R. It may be noted that in the limiting case, i.e., for the dust less cold ion plasma i. e., considering $n_d=0,\ \gamma=1,$ $\alpha=0, and\,\sigma=0$ the above expressions of P, Q and R reduce to earlier case of Yashvir et al. (1985).

Stability analysis and discussion

The linear stability analysis by Nishikawa and Liu (1976) shows that the wave is stable, if the product of the coefficients of dispersive term P and nonlinear term Q is negative i.e., PQ < 0 and for $PQ>0$

the wave becomes unstable for modulational wave number $\tilde{K}_m$ in the region

$$0 < \tilde{K}_m^2 < \frac{2Q|a_0|^2}{P} \tag{32}$$

The maximum growth rate is obtained for

$$\tilde{K}_m = \pm\left(\frac{Q}{P}|a_0|^2\right)^{1/2} \tag{33}$$

and has a value

$$y_m = |Q||a_0|^2 \tag{34}$$

the modulational wave number $\tilde{K}_m$ is assumed to be less than the carrier wave number k and $a_0 = |\tilde{n}|/n_0$ is the normalized carrier wave amplitude.

Now to investigate the instability, we have numerically evaluated the value of PQ for given set of parameters for different values of wave number and plotted in Fig. (1). To depict the variation of PQ with respect to wave number k for different values of ion temperature ratio (σ), taking other plasma parameters fixed as ($M_d = 1\times10^{-5}$; $\alpha = 1\times10^{-3}$; $|\varepsilon_z| = 1\times10^4$; $\mu = 0.3$; and $\beta = 0.3$). The figure clearly shows that there exist two critical values of wave numbers i.e., k_{c1} and k_{c2}, which decide the range of k for which the wave would be modulationally unstable. It is found that the wave would be modulationally unstable for the values of k lying in the range

($0 < k < k_{c1}$) and for $k > k_{c2}$. It is also found that due to the presence of dust the wave becomes modulationally unstable for those lower values of k for which the wave was perfectly stable in the absence of dust. The effect of ion temperature ratio σ on the instability of the waves has been shown in Fig. (1), as the value of σ is increased, the values of k_{c1} increases rapidly for both the polarity of dust charges, however k_{c2} remains the same, at the same time the region of instability increases with increase in σ. The shift of k_{c1} towards higher value of k with increase in σ as shown in enlarged Fig. (1b) clearly establishes that the waves are unstable for higher wave number i.e., the region of instability in the lower range of k increases with increase in σ, however there is no effect on k_{c2}. It is also found that the unstable region in lower side increases by changing the polarity of the dust charge from negative to positive as the value of k_{c1} for a fixed value of σ is higher for positively charged dust as compared to negatively charged dust. At the same time the value of k_{c2} has no effect of the polarity of the charge on dust particle. Thus it may be concluded that region of instability increases by changing the polarity of dust particles from negative to positive.

In Fig. (2), the effect of cold electron concentration μ on the instability of dust ion acoustic wave has been investigated. Our investigation from the curve of PQ with respect to the wave number k shows that increase in cold electron concentration has no effect on the value of k_{c1}, however the value of k_{c2} decreases. Therefore the region of instability increases with increase in μ. From Fig. (2) and the enlarged region shown in Figs. (2a & 2b) for the lower and higher values of k, it can be seen that the value of k_{c1} is higher for the positively charged dust as compared to negatively charged dust, at the same time it may easily be noticed that the value of k_{c2} does not change with polarity of dust particle but decreases with increase in μ. Therefore it may be concluded that the unstable region in presence of the positively charged dust is more as compared to negatively charged dust keeping μ constant and for the same charged polarity, the instability increases with increase in μ.

In Fig. 3, the variation of PQ with respect to wave number k for different values of temperature ratio of two electron species (β) with fixed plasma parameters as ($M_d = 1\times10^{-5}$; $\alpha = 1\times 10^{-3}$; $|\varepsilon_z| = 1\times10^4$; and $\mu = 0.3$) have been plotted. We found from the figure that as the temperature ratio of two species (β) increases, the values of k_{c2} decreases slightly however k_{c1} remains fixed and thereby the region of instability increases. From Fig. (3) and the enlarged regions shown in Figs. (3a & 3b) for the lower and higher values of k, it can be seen that the value of k_{c1} is higher for the positively charged dust as compared to negatively charged dust, at the same time it may also be noticed that the value of k_{c2} does not change with polarity of dust particle but decreases with increase in β. Therefore it may be concluded that the stable region in presence of the positively charged is less as compared to negatively charged dust keeping β constant and for the same charged polarity, the region of instability increases with increase in β.

In Fig. 4, the variation of PQ with respect to wave number k for different values of mass ratio of ion to dust particle (M_d) keeping all other plasma parameters fixed as ($\alpha = 1\times 10^{-3}$; $|\varepsilon_z| = 1\times10^4$; $\beta = 0.3$ and $\mu = 0.3$). It may be noticed from the figures (4b & 4a) that as the mass ratio of ion to dust particle M_d increases, the values of k_{c2} remains fixed, however the value of k_{c1} decreases (increases) with increase in mass in case of negatively (positively) charged dust particle. Therefore it may be concluded that the region of instability increases (decreases) with increase in mass in presence of positively (negatively) charged dust particles. It is also found that the value of k_{c1} is lower (higher) in case of negatively (positively) charged dust.

In Fig. 5, the variation of PQ with respect to wave number k for different values of charge multiplicity has been shown for the fixed plasma parameters as ($M_d = 1\times10^{-5}$; $\alpha = 1\times10^{-3}$; $\beta = 0.3$; and $\mu = 0.3$). From Figure (5a), it is found that k_{c1} increases (decreases) with decrease (increase) in charge multiplicity ratio ε_z for the negatively (positively) charged dust particle in dusty plasma; however there is no effect on k_{c2} of charge multiplicity.

For the actual existence of instability, the following conditions must also be satisfied in addition to the requirement PQ > 0.

(i) The normalized wave number k must be less than one ($k < K_D$).

(ii) The normalized modulational wave number, corresponding to the maximum growth rate of instability must also be less than one ($\widetilde{K}_m < K_D$).

(iii) The maximum growth rate must be greater than the Landau damping rate ($y_m = |Q||a_0|^2 > \gamma_i$).

We have done calculation for the maximum growth rate (y_m) and modulational wave number ($\widetilde{K}_m$) for the two given values of k lies in the unstable regions, one below the critical value k_{c1} in the small wave number region and the other above the critical value k_{c2} for large amplitude (a_0 = 0.1) and small amplitude (a_0 = 0.01). The calculated values for different ion temperature ratio, in case of positive and negative charge multiplicity, for the small value less than k_{c1} are tabulated in Table 1A and 1B respectively. The Table clearly shows the stable and unstable regions. For large values of k above k_{c2} for the same plasma parameters have been shown in Table 2A and 2B. For these given values of k, we also have PQ > 0, in all cases. The calculations are made for two different values of carrier amplitude a_0 and we have assumed $\gamma_i \simeq 10^{-3}\omega_{pi}$.

In Table 3A and 3B the variation of $\widetilde{K}_m$ and y_m with cold electron concentration (μ) has been tabulated for both the polarities of dust species in case of wave number chosen below critical value k_{c1}. Similar calculations done in case of large wave number region above k_{c2} are presented in Table

4A and 4B, it is found that in small wave number region the wave becomes unstable for amplitude above $a_0 = 0.05$. We have performed similar calculations for the other set of plasma parameters. Unstable and stable regions are shown in the tables.

Conclusions

Our main conclusions are as follows:

(i) It is found that for given set of parameters there exist two critical values of wave number (k); i. e., k_{c1} and k_{c2} which decide the modulational instability. It is also found that the wave would be modulationally unstable for the values of k lying in the range ($0 < k < k_{c1}$) and for $k > k_{c2}$.

(ii) It is also investigated that the presence of dust particles drastically affect the unstable region. It is also found that dust ion acoustic wave becomes modulationally unstable for those values of, for which it was perfectly stable in the absence of dust.

(iii) The instable region in presence of the positively charged dust is more as compared to negatively charged dust keeping μ constant and for the same charged polarity, the instability increases with increase in μ.

(iv) It is found that the unstable region in presence of the positively charged is large as compared to negatively charged dust keeping β constant and for the same charged polarity, the unstable region increases with increase in β.

(v) It is found that the region of instability increases (decreases) with increase in mass in presence of positively (negatively) charged dust particles and that the value of k_{c1} is lower (higher) in case of negatively (positively) charged dust.

(vi) It is found that k_{c1} increases (decreases) with decrease (increase) in charge multiplicity ratio ε_z for the negatively (positively) charged dust particle in dusty plasma; however there is no effect on k_{c2} of charge multiplicity. Therefore it may be concluded that the region of instability increases (decreases) in case of positively (negatively) charged dust particles with increase in charge multiplicity ratio.

The results obtained in the study may be useful to explain the stable propagation of dust ion acoustic wave in the astrophysical environments such as auroral plasma, magnetospheric and laboratory plasma where dust particle with two temperature electrons distributions, along with warm ions are present.

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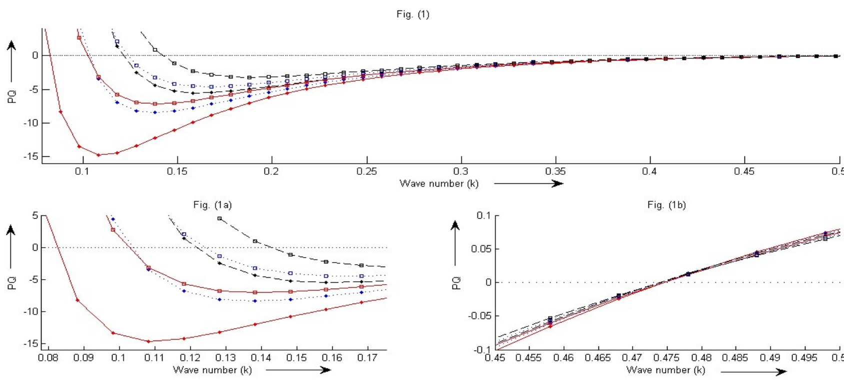

Figure 1. Variation of PQ with the wave number k for different values of temperature ratio of the two species of electrons (σ) = 0.004 (solid line), 0.006 (dotted line) and 0.008 (dashed line) for the fixed plasma parameters as ($\varepsilon_z = 1\times 10^4$; $M_d = 1\times 10^{-5}$; $\alpha = 0.001$; $\mu = 0.3$ and $\beta = 0.3$). The negative (positive) polarity of dust particles are shown by stars and squares respectively. In Fig. (1a) and (1b) enlarged region near k_{c1} and k_{c2} has been shown.

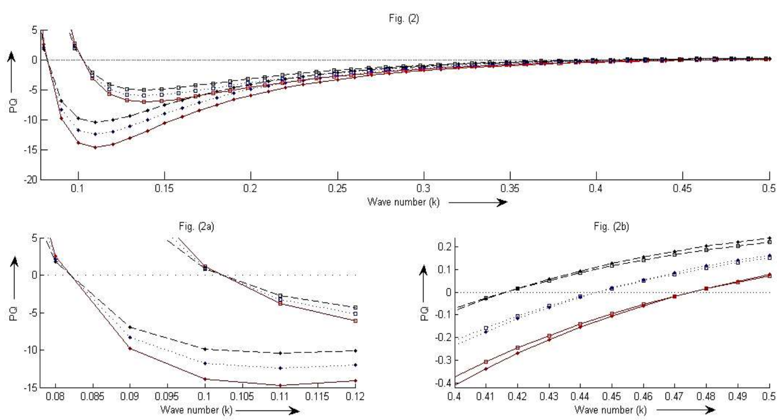

Figure 2. Variation of PQ with the wave number k for different values of cold electron concentration (μ) = 0.3 (solid line), 0.35 (dotted line) and 0.4 (dashed line) for the fixed plasma parameters as ($M_d = 1\times 10^{-5}$; $\alpha = 0.3$; $\varepsilon_z = 1\times 10^4$; $\beta = 0.3$ and $\sigma = 0.004$). The negative

(positive) polarity of dust particles are shown by stars and squares respectively. In Fig. (2a) and (2b) enlarged region near k_{c1} and k_{c2} has been shown.

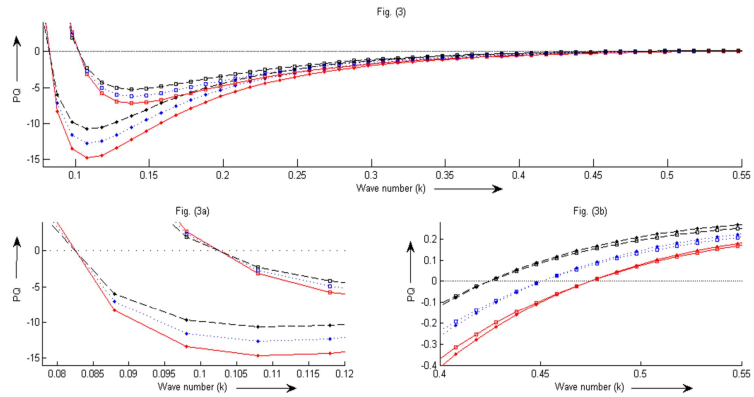

Figure 3. Variation of PQ with the wave number k for different values of ion temperature ratio (β) = 0.3 (solid line), 0.35 (dotted line) and 0.4(red dashed line) for the fixed plasma parameters as ($M_d = 1\times10^{-5}$; $\alpha = 0.001$; $\varepsilon_z = 1\times10^{4}$; and $\mu = 0.3$). The negative (positive) polarity of dust particles are shown by stars and squares respectively. In Fig. (3a) and (3b) enlarged region near k_{c1} and k_{c2} has been shown.

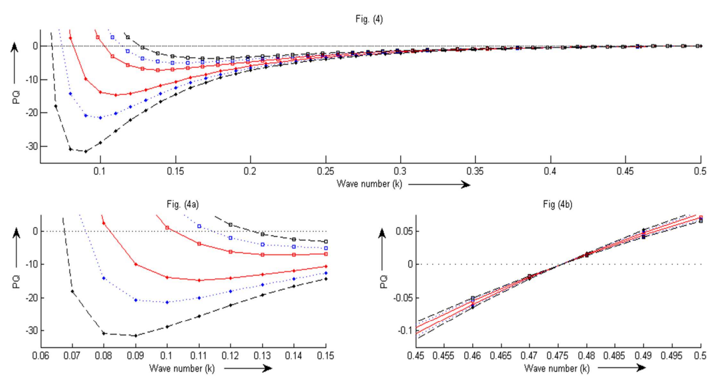

Figure 4. Variation of PQ with the wave number k for different values of ion to dust particle mass ratio $(M_d) = 1\times10^{-6}$ (solid line), 2×10^{-6} (dotted line) and 3×10^{-6} (dashed line) for the fixed plasma parameters as ($\alpha = 0.001$; $\varepsilon_z = 1\times10^4$; $\sigma = 0.004$; $\beta = 0.3$ and $\mu = 0.3$). The negative (positive) polarity of dust particles are shown by stars and squares respectively. In Fig. (4a) and (4b) enlarged region near k_{c1} and k_{c2} has been shown.

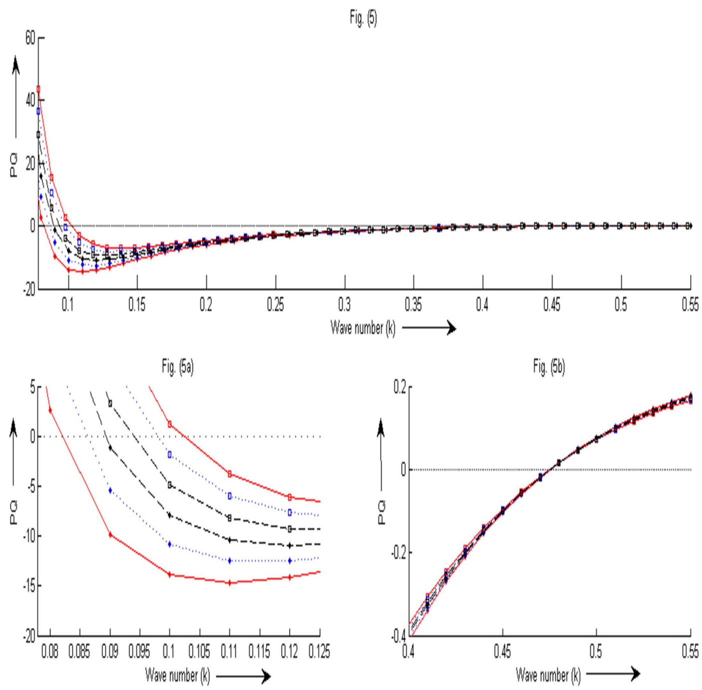

Figure 5. Variation of PQ with the wave number k for different values of charge multiplicity ratio $(\varepsilon_z) = 1\times10^{-5}$ (solid line), 6×10^4 (dotted line) and 2×10^4 (dashed line) for the fixed p lasma parameters as ($\alpha = 0.001$; $M_d = 1\times10^{-5}$; $\sigma = 0.004$; $\beta = 0.3$ and $\mu = 0.3$). The negative (positive) polarity of dust particles are shown by stars and squares respectively. In Fig. (5a) and (5b) enlarged region near k_{c1} and k_{c2} has been shown.

TABLE – 1A

	Carrier amplitude (a_0 =0.005)			Carrier amplitude (a_0 =0.01)		
σ	$\widetilde{K}_m$	y_m	Inference	$\widetilde{K}_m$	y_m	Inference
0.004	5.56×10^{-2}	8.03×10^{-4}	Stable	1.11×10^{-1}	3.2×10^{-3}	Unstable
0.005	5.27×10^{-2}	1.20×10^{-3}	(Marginally	1.06×10^{-1}	4.6×10^{-3}	Unstable
0.006	5.48×10^{-2}	1.70×10^{-3}	Unstable	1.11×10^{-1}	6.9×10^{-3}	Unstable
0.007	6.12×10^{-2}	2.70×10^{-3}	Unstable	1.22×10^{-1}	1.1×10^{-2}	Unstable
0.008	7.59×10^{-2}	5.10×10^{-3}	Unstable	1.56×10^{-1}	2.0×10^{-2}	Unstable

TABLE 1A. Variation of the modulational wave number ($\widetilde{K}_m$) and the maximum instability growth rate (y_m) with ion to electron temperature ratio (σ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\varepsilon_z = 1\times10^{4}$; $\mu = 0.3$; $\beta = 0.3$ and $k = 0.08$.

TABLE – 1B

	Carrier amplitude (a_0 =0.005)			Carrier amplitude (a_0 =0.01)		
σ	$\widetilde{K}_m$	y_m	Inference	$\widetilde{K}_m$	y_m	Inference
0.004	2.1×10^{-1}	8.07×10^{-4}	Stable	4.21×10^{-1}	3.2×10^{-3}	Unstable
0.005	8.14×10^{-2}	1.2×10^{-3}	(Marginally	1.63×10^{-1}	4.7×10^{-3}	Unstable
0.006	7.21×10^{-2}	1.7×10^{-3}	Unstable	1.44×10^{-1}	6.9×10^{-3}	Unstable
0.007	7.51×10^{-2}	2.8×10^{-3}	Stable	1.50×10^{-1}	1.1×10^{-2}	Unstable
0.008	8.97×10^{-2}	5.2×10^{-3}	Unstable	1.79×10^{-1}	2.1×10^{-2}	Unstable

TABLE 1B. Variation of the modulational wave number ($\tilde{K}_m$) and the maximum instability growth rate (y_m) with ion to electron temperature ratio (σ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\varepsilon_z = -1\times10^4$; $\mu = 0.3$; $\beta = 0.3$ and $k = 0.08$.

TABLE – 2A

	Carrier amplitude (a_0 =0.01)			Carrier amplitude (a_0 =0.1)		
σ	$\tilde{K}_m$	y_m	Inference	$\tilde{K}_m$	y_m	Inference
0.004	1.80×10^{-2}	8.52×10^{-5}	Stable	1.8×10^{-1}	8.5×10^{-3}	Unstable
0.005	1.82×10^{-2}	8.53×10^{-5}	Stable	1.82×10^{-1}	8.5×10^{-3}	Unstable
0.006	1.84×10^{-2}	8.54×10^{-5}	Stable	1.84×10^{-1}	8.5×10^{-3}	Unstable
0.007	1.86×10^{-2}	8.55×10^{-5}	Stable	1.86×10^{-1}	8.6×10^{-3}	Unstable
0.008	1.89×10^{-2}	8.57×10^{-5}	Stable	1.89×10^{-1}	8.6×10^{-3}	Unstable

TABLE 2A. Variation of the modulational wave number ($\tilde{K}_m$) and the maximum instability growth rate (y_m) with ion to electron temperature ratio (σ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\varepsilon_z = 1\times10^4$; $\mu = 0.3$; $\beta = 0.3$ and $k = 0.48$.

TABLE – 2B

	Carrier amplitude (a_0 =0.01)			Carrier amplitude (a_0 =0.1)		
σ	$\tilde{K}_m$	y_m	Inference	$\tilde{K}_m$	y_m	Inference
0.004	1.73×10^{-2}	8.521×10^{-5}	Stable	1.8×10^{-1}	8.5×10^{-3}	Unstable
0.005	1.75×10^{-2}	8.532×10^{-5}	Stable	1.82×10^{-1}	8.5×10^{-3}	Unstable
0.006	1.77×10^{-2}	8.544×10^{-5}	Stable	1.84×10^{-1}	8.5×10^{-3}	Unstable
0.007	1.79×10^{-2}	8.556×10^{-5}	Stable	1.86×10^{-1}	8.6×10^{-3}	Unstable
0.008	1.81×10^{-2}	8.607×10^{-5}	Stable	1.89×10^{-1}	8.6×10^{-3}	Unstable

TABLE 2B. Variation of the modulational wave number ($\tilde{K}_m$) and the maximum instability growth rate (y_m) with ion to electron temperature ratio (σ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\varepsilon_z = -1\times10^4$; $\mu = 0.3$; $\beta = 0.3$ and $k = 0.48$.

TABLE – 3A

	Carrier amplitude (a_0 =0.005)			Carrier amplitude (a_0 =0.01)		
μ	$\tilde{K}_m$	y_m	Inference	$\tilde{K}_m$	y_m	Inference
0.300	5.56×10^{-2}	8.03×10^{-4}	Stable	1.11×10^{-1}	3.2×10^{-3}	Unstable
0.325	4.51×10^{-2}	5.3×10^{-4}	Stable	9.03×10^{-2}	2.1×10^{-3}	Unstable
0.350	3.27×10^{-2}	2.79×10^{-4}	Stable	6.54×10^{-2}	1.1×10^{-3}	(Marginally unstable)
0. 375	1.31×10^{-2}	4.49×10^{-4}	Stable	2.63×10^{-2}	1.79×10^{-4}	Stable
0.400	2.58×10^{-2}	1.73×10^{-4}	Stable	5.16×10^{-2}	6.94×10^{-4}	Stable

TABLE 3A. Variation of the modulational wave number ($\tilde{K}_m$) and the maximum instability growth rate (y_m) with cold electron concentration (μ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\beta = 0.3$; $\varepsilon_z = 1\times10^4$; $\sigma = 0.004$ and $k = 0.08$.

TABLE – 3B

	Carrier amplitude (a_0 =0.005)			Carrier amplitude (a_0 =0.01)		
μ	$\tilde{K}_m$	y_m	Inference	$\tilde{K}_m$	y_m	Inference
0.300	2.1×10^{-1}	8.07×10^{-4}	Stable	4.21×10^{-1}	3.2×10^{-3}	Unstable
0.325	1.71×10^{-1}	5.34×10^{-4}	Stable	3.42×10^{-1}	2.1×10^{-3}	Unstable
0.350	1.24×10^{-2}	2.81×10^{-4}	Stable	2.48×10^{-1}	1.1×10^{-3}	(Marginally unstable)
0. 375	5.07×10^{-2}	4.69×10^{-5}	Stable	1.02×10^{-1}	1.88×10^{-3}	Unstable
0.400	9.71×10^{-2}	1.72×10^{-4}	Stable	1.94×10^{-1}	6.87×10^{-4}	Stable

TABLE 3B. Variation of the modulational wave number ($\widetilde{K}_m$) and the maximum instability growth rate (y_m) with cold electron concentration (μ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\beta = 0.3$; $\varepsilon_z = -1\times10^4$; $\sigma = 0.004$ and $k = 0.08$.

TABLE – 4A

	Carrier amplitude (a_0 =0.01)			Carrier amplitude (a_0 =0.1)		
μ	$\widetilde{K}_m$	y_m	Inference	$\widetilde{K}_m$	y_m	Inference
0.300	1.85×10^{-2}	8.66×10^{-5}	Stable	1.85×10^{-1}	8.7×10^{-3}	Unstable
0.325	2.01×10^{-2}	1.03×10^{-4}	Stable	2.01×10^{-1}	1.03×10^{-2}	Unstable
0.350	2.15×10^{-2}	1.17×10^{-4}	Stable	2.15×10^{-1}	1.17×10^{-2}	Unstable
0. 375	2.27×10^{-2}	1.31×10^{-4}	Stable	2.27×10^{-1}	1.31×10^{-2}	Unstable
0.400	2.38×10^{-2}	1.43×10^{-4}	Stable	2.38×10^{-1}	1.43×10^{-2}	Unstable

TABLE 4A. Variation of the modulational wave number ($\widetilde{K}_m$) and the maximum instability growth rate (y_m) with cold electron concentration (μ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\beta = 0.3$; $\varepsilon_z = 1\times10^4$; $\sigma = 0.004$ and $k = 0.5$.

TABLE – 4B

	Carrier amplitude (a_0 =0.01)			Carrier amplitude (a_0 =0.1)		
μ	$\widetilde{K}_m$	y_m	Inference	$\widetilde{K}_m$	y_m	Inference
0.300	1.78×10^{-2}	8.66×10^{-5}	Stable	1.78×10^{-1}	8.7×10^{-3}	Unstable
0.325	1.93×10^{-2}	1.03×10^{-4}	Stable	1.93×10^{-1}	1.03×10^{-2}	Unstable
0.350	2.07×10^{-2}	1.17×10^{-4}	Stable	2.07×10^{-1}	1.17×10^{-2}	Unstable
0. 375	2.18×10^{-2}	1.31×10^{-4}	Stable	2.18×10^{-1}	1.31×10^{-2}	Unstable
0.400	2.29×10^{-2}	1.44×10^{-4}	Stable	2.29×10^{-1}	1.44×10^{-2}	Unstable

TABLE 4B. Variation of the modulational wave number ($\widetilde{K}_m$) and the maximum instability growth rate (y_m) with cold electron concentration (μ) for the fixed plasma parameters as $M_d = 1\times10^{-6}$; $\alpha = 0.001$; $\beta = 0.3$; $\varepsilon_z = -1\times10^4$; $\sigma = 0.004$ and $k = 0.5$.