A Study on the Significance of the Matrix and Linear Algebra in Mathematics

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ABSTRACT:

Basic linear systems may be understood via the use of matrices and vectors in linear algebra. The field of mathematics known as linear algebra studies vectors, vector spaces (also known as linear spaces), linear mappings (also known as linear transformations), and linear equation systems. Linear algebra is often used in both abstract algebra and functional analysis as vector spaces constitute a major topic in contemporary mathematics. In addition to being generalized in operator theory, linear algebra also has a concrete representation in analytic geometry. Since linear models may often be used to approximate nonlinear ones, it has many uses in both the scientific and social sciences.

Keywords: Linear Algebra, Matrix, Linear Equation, n- Tuples, Vectors, Linear Spaces

I. INTRODUCTION

The study of vectors in Cartesian 2-space and 3-space served as the foundation for linear algebra. In this context, a vector is a directed line segment that is distinguished by both its direction and magnitude, which are expressed in terms of length.

Physical things like forces may be represented by vectors, which can then be multiplied by scalars and joined to one another to create the first real vector space example. Spaces of arbitrary or infinite dimensions are now taken into consideration by modern linear algebra. An n-space is a vector space of dimension n. These higher dimensional spaces may benefit from the majority of the practical findings from 2- and 3-space. Vectors or n-tuples are helpful for expressing data, despite the fact that they are difficult for humans to visualize in n-space. In this paradigm, data may be effectively summarized and manipulated since vectors, as n-tuples, are ordered lists of n components. For instance, in economics, the Gross National Product of eight nations may be represented using, say, eight-dimensional vectors or eight-tuples. By using a vector (v1, v2, v3, v4, v5, v6, v7, v8) with each country's GNP in its proper position, one can choose to display the GNP of eight countries for a specific year, where the order of the countries is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, Australia). As a totally abstract idea about which theorems are established, a vector space (also known as a linear space) belongs to abstract algebra and fits in well with this field. The ring of linear mappings of a vector space and the group of invertible linear maps or matrices are two remarkable instances of this. Additionally, linear algebra is crucial to analysis, particularly for describing higher order derivatives in vector analysis and studying tensor products and alternating maps.

II. ESSENTIAL OVERVIEW

The study of vectors in cartesian 2-space and 3-space served as the foundation for linear algebra. Here, a vector is a segment of a directed line that is distinguished by both its direction and magnitude, which are also referred to as length or norm. An exception is the zero vector, which has no direction nor magnitude. This is the first example of a real vector space, where "scalars" (in this instance, real numbers) and "vectors" are distinguished. Vectors may be used to represent physical phenomena like forces, and they can be multiplied by scalars and added to one another.

Spaces of arbitrary or infinite size are now taken into consideration in modern linear algebra. An n-space is a vector space of dimension n. It is possible to apply the majority of the practical findings from 2- and 3-space to these higher dimensional areas. Vectors or n-tuples are valuable for expressing data, despite the fact that they are difficult for humans to visualize in n-space. This framework allows for the efficient summarization and manipulation of data since vectors, like n-tuples, are composed of n ordered components. For instance, in economics, the gross national product of eight nations may be represented using, say, eight-dimensional vectors or eight-tuples. By using a vector (v1, v2, v3, v4, v5, v6, v7, v8) where each country's GNP is in its respective position, one can choose to display the GNP of eight countries for a specific year, where the countries' order is specified, for example, (United States, United Kingdom, Armenia, Germany, Brazil, India, Japan, Bangladesh).

III. SOME USEFUL THEOREMS

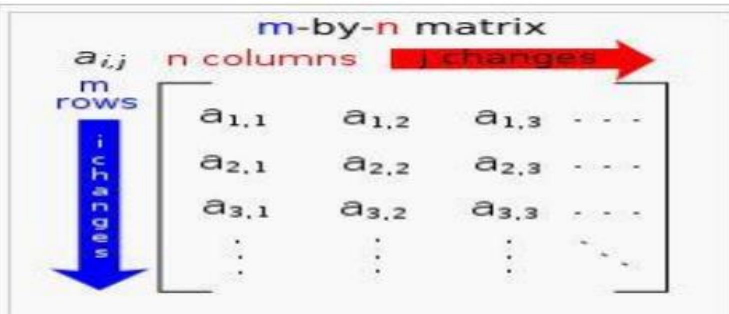
- Every vector space has a basis [1].
- Any two bases of the same vector space have the same cardinality; equivalently, the dimension of a vector space is well-defined.
- A matrix is invertible if and only if its determinant is nonzero.
- A matrix is invertible if and only if the linear map represented by the matrix is an isomorphism.
- If a square matrix has a left inverse or a right inverse then it is invertible (see invertible matrix for other equivalent statements).
- A matrix is positive semidefinite if and only if each of its eigen values is greater than or equal to zero.
- A matrix is positive definite if and only if each of its eigen values is greater than zero.
- An n×n matrix is diagonalizable (i.e. there exists an invertible matrix P and a diagonal matrix D such that A = PDP-1) if and only if it has n linearly independent eigenvectors.
- The spectral theorem states that a matrix is orthogonally diagonalizable if and only if it is symmetric.

For more information regarding the invertability of a matrix, consult the invertable matrix article.

IV. LINEAR EQUATION

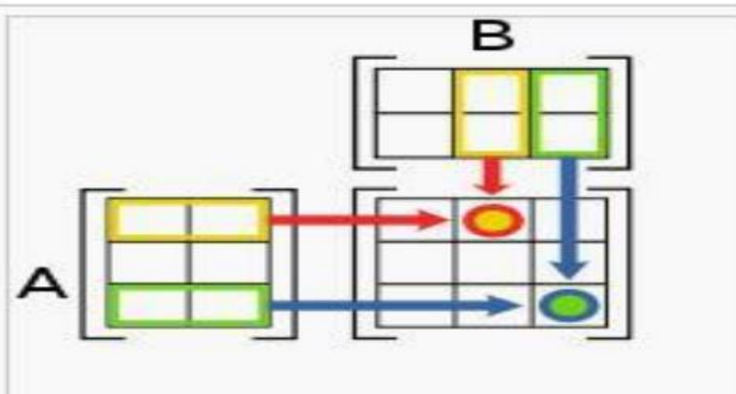
When every term in an algebraic equation is either a constant or the product of a constant and one variable (to the first power of that variable), the equation is said to be linear. One or more variables may be included in linear equations. The majority of mathematical subfields, particularly applied mathematics, use linear equations extensively. Although they naturally occur when modeling a variety of processes, they are especially helpful since many nonlinear equations may be simplified to linear equations by supposing that values of interest differ from a "background" condition only little. Exponents are not used in linear equations. The situation when one looks for the actual solutions to a single equation is examined in this article. Its whole material is applicable to complicated solutions as well as, more broadly, linear equations in any subject that have coefficients and solutions.

V. MATRIX



Organization of a matrix

VI. MATRIX MULTIPLICATION, LINEAR EQUATIONS AND LINEAR TRANSFORMATIONS



Schematic depiction of the matrix product **AB** of two matrices **A** and **B**.

Only when the number of rows in the right matrix equals the number of columns in the left matrix can two matrices be multiplied. The m-by-p matrix whose entries are provided by: is the matrix product of A and B, where A is an m-by-n matrix and B is an n-by-p matrix.

$$[\mathrm{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \dots + A_{i,n}B_{n,j} = \sum_{r=1} A_{i,r}B_{r,j},$$

where 1 ≤ i ≤ m and 1 ≤ j ≤ p. [5] For example (the underlined entry 1 in the product is calculated as the product 1 · 1 + 0 · 1 +2·0=1):

$$\begin{bmatrix} \underline{1} & \underline{0} & \underline{2} \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & \underline{1} \\ 2 & \underline{1} \\ 1 & \underline{0} \end{bmatrix} = \begin{bmatrix} 5 & \underline{1} \\ 4 & 2 \end{bmatrix}.$$

Matrix multiplication satisfies the rules (AB)C = A(BC) (associativity), and (A+B)C = AC+BC as well as C(A+B) = CA+CB (left and right distributivity), whenever the size of the matrices is such that the various products are defined.

The product AB may be defined without BA being defined, namely if A and B are m-by-n and n-by-k matrices, respectively, and m ≠ k. Even if both products are defined, they need not be equal, i.e. generally one has AB ≠ BA, i.e., matrix multiplication is not commutative, in marked contrast to (rational, real, or complex) numbers whose product is independent of the order of the factors.

Linear Equations

A specific instance of matrix multiplication is closely related to linear equations: if A is an m-by-n matrix and x represents a column vector (i.e., n×1-matrix) containing n variables x1, x2,..., xn, then the matrix equation

Ax = b, where b is some m×1-column vector, is equivalent to the system of linear equations

$$A_{1,1}x_1 + A_{1,2}x_2 + \dots + A_{1,n}x_n = b_1$$
$$A_{m,1}x_1 + A_{m,2}x_2 + \dots + A_{m,n}x_n = b_m .$$

In this manner, many linear equations, or systems of linear equations, may be written and handled compactly using matrices.

Matrices and matrix multiplication reveal their essential features when related to linear transformations, also known as linear maps. A real m-by-n matrix A gives rise to a linear transformation R n → R m mapping each vector x in R n to the (matrix) product Ax, which is a vector in R m. Conversely, each linear transformation f: R n → R m arises from a unique m-by-n matrix A: explicitly, the (i, j)-entry of A is the ith coordinate of f(ej), where ej = (0,,0,1,0,...,0) is the unit vector with 1 in the jth position and 0 elsewhere. The matrix A is said to represent the linear map f, and A is called the transformation matrix off. The following table shows a number of 2-by-2 matrices with the associated linear maps of R 2 . The blue original is mapped to the green grid and shapes, the origin (0,0) is marked with a black point.

VII. CONCLUSIONS

In contemporary physics, linear transformations and the corresponding symmetries are essential. Matrix analysis is used in many aspects of chemistry, especially since quantum theory was introduced to explain spectroscopy and molecular bonding. Here, we are introducing a mathematical study on linear algebra and matrices.An algebraic equation is said to be linear if every term is either a constant or the result of a constant multiplied by one variable (the first power of). One or more variables may be included in linear equations. The study of vectors, vector spaces (also known as linear spaces), linear mappings (also known as linear transformations), and systems of linear equations is the focus of the mathematical field known as linear algebra.

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