Korteweg-de Vries equation on ion-acoustic Solitons in plasmas With Two Temperature Superthermal Electrons

*S.K. Jain
**J.K. Chawla

Abstract

Small amplitude ion-acoustic compressive solitons in plasma consisting of ions, positrons and superthermal electrons have been studied. For this purpose, the hydrodynamics equations for ions, positrons, superthermal electron density distribution along with the Possion's equation are used. The reductive perturbation method is employed to derive the Korteweg de Vries (KdV) equation. Variation of amplitude and width for the compressive solitons are graphically represented for different values of positrons concentrations, spectral index and ionic temperature ratio. The amplitude of the compressive solitons increases with increase in positron concentrations and on increasing the ionic temperature ratio. The study of such solitons may be useful for the critical understanding of space, where superthermal electrons exist along with positrons and ions.

Introduction

Ion-acoustic rarefactive solitons in a two-electron-temperature plasma exist only for certain density and temperature ratios of the hot to cold electron species. (Buti 1980; Nishihara and Tajiri 1981; Dash and Buti 1981; Krokhin and Tsybenko 1986). There is broad agreement between the results derived with fluid theory and experimental results (Nishida and Nagasawa 1986). However, experimental studies of the ion-acoustic rarefactive solitons show that there are discrepancies between theoretical predictions and experimental observations (Nishida and Nagasawa 1986). Sayal and Sharma (1990) have studied ion-acoustic rarefactive solitons in a two-electron-temperature plasma considering kinetic effects of electrons and using the fluid equations for cold ions. A large number of authors few to cite (Baboolal et al. 1990; Yadav and Sharma 1990; Sayal et al. 1993; Ghosh et al. 1996; Mishra et al. 2007; Bharuthram et al. 2008; Baluku and Hellberg 2012; Jain and Mishra 2013b; Chawla and Mishra 2012) have studied nonlinear waves in two-electron temperature plasma. Ghosh et al. (1996) reported that with large Mach number, there exists only rarefactive solitons in warm electron-ion plasma with two species of electrons. Mishra et al. [2012] have studied the ion-acoustic solitons in a warm plasma consisting of both polarities of ion species with different masses, concentrations and charge states along with two-electron temperature distributions. They derived a

KdV equation applying reductive-perturbation technique, which admits a soliton solution. It is found that at the critical concentration of negative ions both compressive and rarefactive solitons coexist. They have reported the effect of different parameters on the characteristics of the solitons. Jain and Mishra (2013) studied the ion acoustic solitons in multi-component collisionless plasma consisting of warm adiabatic ions, hot positrons and two-electron temperature distributions separately in thermal equilibrium. They found that due to finite concentration of positrons both subsonic and supersonic rarefactive solitons exist in electron-positron-ion plasma.

Sabry et al. [2009] have studied fully nonlinear ion-acoustic solitary waves in a plasma with cold positive-negative ions and nonthermal electrons following Cairn's [1995] distribution. Arbitrary amplitude ion-acoustic solitons in plasma consisting of cold ions and superthermal electrons have been investigated by Saini et al. [2009]. El-Labany [2013] has investigated nonlinear wave propagation of large amplitude ion-acoustic solitary waves in cold negative ion plasma with soperthermal electrons having Kappa distribution. They found that only compressive solitons exists in the system. Effects of two-temperature superthermal electrons on dust-ion-accoustic solitary waves and double layers in dusty plasma have been studied by Alam et. Al. [2013]. Saini et al. [2014] investigated the stable and unstable ion acoustic solitary waves in a magnetized plasma. Panwar et al. [2014] have studied the oblique propagation of ion acoustic cnoidal waves in two temperature superthermal electrons in magnetized plasma.

Above investigation suggest that plasma with two temperature superthermal distribution has not been studied so far, however superthermal parameters affect drastically the nature and properties of the solitons therefore it becomes interesting to study the properties of small amplitude ion-acoustic solitons in plasma consisting of warm adiabatic ions, hot positrons and superthermal electrons with two temperature distribution.

The aim of present paper is to study the properties of small amplitude ion-acoustic solitons in plasmas with superthermal electrons with two temperature distribution. The paper is organized as follows: in section 2, we have describe the basic equations governing plasma system. Using reductive perturbation method KdV equation is derived with appropriate boundary conditions. In section 3, the exact solutions of the KdV equation has been determined. Next section is devoted to discussion of the results. In last section, we have summarized the conclusions of our investigations.

Basic equations

We consider a collisions, unmagnetized, plasma consisting of warm adiabatic ions, hot positrons and superthermal electrons with two temperature distribution. The governing equations for ion-acoustic waves in the above described plasma are as follows:

$$\partial_t N + \partial_x (NV) = 0 \tag{1}$$

$$\partial_t V + V\partial_x V = -\partial_x \phi - 2\sigma\partial_x N \tag{2}$$

$$\partial_x^2 \phi = n_h + n_c - \alpha n_p - (1-\alpha) N \tag{3}$$

The electrons follow kappa distribution governed by

$$n_c = \nu \left(1 - \frac{\phi}{k_c - 3/2}\right)^{-(k_c - 1/2)}$$

$$n_h = \mu \left(1 - \frac{\sigma_1 \phi}{k_h - 3/2}\right)^{-(k_h - 1/2)}$$

$$n_p = e^{-\gamma\phi} = \left(1 - \gamma\phi + \frac{\gamma^2\phi^2}{2} - \frac{\gamma^3\phi^3}{6} + \ldots\right) \tag{4}$$

Where $\mu = \frac{n_{c0}}{n_{e0}}$, $\nu = \frac{n_{h0}}{n_{e0}}$, $\alpha = \frac{n_{p0}}{n_{e0}}$, $\alpha_1 = \frac{(1-\mu)(2k_c - 1)}{(2k_c - 3)} + \frac{\sigma_1 \mu (2k_h - 1)}{(2k_h - 3)}$,

$\alpha_2 = \frac{(1-\mu)(4k_c^2 - 1)}{2(2k_c - 3)^2} + \frac{\sigma_1^2 \mu (4k_h^2 - 1)}{2(2k_h - 3)^2}$ and $k_{c,h}$ is the spectral index.

In the above equations N, n_p, n_c and n_h denote normalized density of ions, positron, cold and hot electrons respectively. V is normalized fluid velocity of the ions and ϕ is the electric potential. $\gamma = T_p / T_e$, $\sigma = T_i / T_e$ and $\sigma_1 = T_c / T_h$ are the ratio of positron to electron temperature, the ratio of ion to electron temperature and the ratio of cold to hot electron temperature respectively. The space coordinate speed (x) has been normalized in terms of Debye length speed $\lambda_D = (\varepsilon_0 T_e / n_0 e^2)^{1/2}$ and time coordinates by the inverse of ion plasma frequency.

To derive the KdV equation from the basic set of equations, viz. Eqs. (1)-(4), we use reductive perturbation technique, introducing the following stretched coordinates (ξ) and (τ) as:

$$\xi = \varepsilon^{1/2}(x - St) \tag{5a}$$

and

$$\tau = \varepsilon^{3/2} t \tag{5b}$$

Where ε a small parameter and S is the phase velocity of the wave, to be determined later.

Now we expand the field quantities in the following form:

$$\begin{bmatrix} N \\ V \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} N^{(1)} \\ V^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} N^{(2)} \\ V^{(2)} \\ \phi^{(2)} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} N^{(3)} \\ V^{(3)} \\ \phi^{(3)} \end{bmatrix} + \dots \tag{6}$$

On substituting the expansion (6) into Eqns. (1) to (4), using Eqns. (5) and equating terms with the same powers of ε, we obtain a set of equations for each order in ε. The set of equations for the lowest order, i.e., $O(\varepsilon)$ are:

$$-S\partial_\xi N^{(1)} + \partial_\xi V^{(1)} = 0 \tag{7}$$

$$-S\partial_\xi V^{(1)} = -\partial_\xi \phi^{(1)} - 2\sigma\partial_\xi N^{(1)} \tag{8}$$

$$\alpha_1\phi^{(1)} + \alpha\gamma\phi^{(1)} - (1-\alpha)N^{(1)} = 0 \tag{9}$$

Solving the above Eqns. (7) to (9); we find the following first-order solutions:

$$N^{(1)} = \frac{1}{(S^2 - 2\sigma)}\phi^{(1)} \tag{10}$$

$$V^{(1)} = \frac{S}{(S^2 - 2\sigma)}\phi^{(1)} \tag{11}$$

On using equations (10) and (11) in Poisson equation (9) to the lowest-order, i.e., $O(\varepsilon)$, we get the following linear relation

$$S^2 = 2\sigma + \frac{(1-\alpha)}{(\alpha_1 + \alpha\gamma)} \tag{12}$$

Now taking the next higher order of Eqns. (1) and (4)

$$-S\partial_\xi N^{(2)} + \partial_\tau N^{(1)} + \partial_\xi (V^{(2)} + N^{(1)}V^{(1)}) = 0 \tag{13}$$

$$-S\partial_\xi V^{(2)} + \partial_\tau V^{(1)} + V^{(1)}\partial_\xi V^{(1)} = -\partial_\xi \phi^{(2)} - 2\sigma\partial_\xi N^{(2)} \tag{14}$$

$$\partial_\xi^2 \phi^{(1)} = (\alpha_1 + \alpha\gamma)\phi^{(2)} + \left(\alpha_2 - \frac{\alpha\gamma^2}{2}\right)\phi^{(1)^2} - (1-\alpha)\frac{1}{(1-\alpha\varepsilon_Z)} N^{(2)} \tag{15}$$

Using Eqns. (13) and (14), we obtain

$$\partial_\xi N^{(2)} = \left[\frac{2S}{(S^2 - 2\sigma)^2}\partial_\tau \phi^{(1)} + \frac{3S^2}{(S^2 - 2\sigma)^3}\phi^{(1)}\partial_\xi \phi^{(1)} + \partial_\xi \phi^{(2)}\right] \tag{16}$$

Differentiating Eqn. (15) with respect to ξ and Using equations (13) and (14)

$$\partial_\xi^3 \phi^{(1)} + \left(\frac{3(1-\alpha)S^2}{(S^2 - 2\sigma)^3} - 2\left(\alpha_2 - \frac{\alpha\gamma^2}{2}\right)\right)\phi^{(1)}\partial_\xi \phi^{(1)} + \frac{2S(1-\alpha)}{(S^2 - 2\sigma)^2}\partial_\tau \phi^{(1)} = 0 \tag{17}$$

From the above Eqn. (17), we find out the following KdV equation:

$$\partial_\tau \phi^{(1)} + PQ\phi^{(1)}\partial_\xi \phi^{(1)} + \frac{1}{2}P\partial_\xi^3 \phi^{(1)} = 0 \tag{18}$$

$$P = \frac{(S^2 - 2\sigma^2)^2}{S(1-\alpha)} \tag{19}$$

and

$$Q = \frac{1}{2}\left(\frac{3S^2(1-\alpha)}{(S^2 - 2\sigma)^3} - 2\left(\alpha_2 - \frac{\alpha\gamma^2}{2}\right)\right) \tag{20}$$

SOLUTION OF KdV EQUATION

For the steady state solution of the KdV Eqns. (18), we consider

$$\zeta = \xi - u\tau. \tag{21}$$

Where u is a constant velocity.

Using Eqn. (21) in (20) integrating with respect to η, we obtain

$$\frac{1}{2}\left(d_{\zeta}\phi\right)^{2}+V(\phi)=0 \tag{22}$$

Where ϕ is used in place of $\phi^{(1)}$ for convenience and $V(\phi)$ is the Sagdeev potential, which is given by

$$V(\phi)=\frac{2}{P}\left(\frac{1}{6}PQ\phi^{3}-\frac{1}{2}u\phi^{2}\right) \tag{23}$$

In the derivation of Eqn. (23) we have used the following boundary conditions. As $\zeta\to\pm\infty$, ϕ, $d_{\zeta}\phi$, and $d_{\zeta}^{2}\phi\to 0$. However, for the soliton solution, the Sagdeev potential $V(\phi)$ should be negative between $\phi=0$ and $\phi=\phi_m$, where ϕ_m is some maximum or minimum value of potential for the compressive and rarefactive solitons, respectively. The following boundary conditions on the Sagdeev potential should be satisfied

$V(\phi)=0$ at $\phi=0$ and $\phi=\phi_m$, (24a)

$V^{'}(\phi)=0$ at $\phi=0$, (24b)

$V^{'}(\phi)>0$ at $\phi=\phi_m$ for compressive soliton, (24c)

$V^{'}(\phi)<0$ at $\phi=\phi_m$ for rarefactive soliton. (24d)

The soliton solution of Eqn. (22) is given by

$$\phi=\phi_m\sec h^{2}\left[W^{-1}(\xi-u\tau)\right] \tag{25}$$

Where the amplitude (ϕ_m) and width (W) are given by

$$\phi_m=\frac{3u}{PQ} \tag{26}$$

and

$$W=\sqrt{\frac{2P}{u}} \tag{27}$$

Result and discussion

To investigate the existence regions and nature of the ion-acoustic solitary wave in plasma with superthermal electrons, we use numerical calculations for different set of plasma parameters. In Figure (1) variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of ion temperature ratio σ = 0.01 (black solid line), 0.02 (red dashed line) and 0.03 (blue dotted line) for the fixed set of plasma parameters k_c = 0.0001, k_h = 0.0015, α = 0.4, γ = 0.3, σ_1 = 0.1, υ = 0.01 and constant velocity (u) = 0.01 have been investigated. It is found that the amplitude of ion-acoustic soliton decreases with an increase in σ however width of the ion-acoustic soliton increases.

In Fig. (2) variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of ion concentrations α = 0.4 (black solid line), 0.401 (red dashed line) and 0.402 (blue dotted line) for the fixed set of plasma parameters k_c = 0.0001, k_h = 0.0015, σ = 0.01, γ = 0.3, σ_1 = 0.1, υ = 0.01 and constant velocity (u) = 0.01. It is found that the amplitude of ion-acoustic soliton increases with an increase in α, however width of the ion-acoustic soliton decreases.

In Fig. (3) variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for the variation of spectral index of hot superthermal electrons of k_h = 0.001 (red dashed line) and 0.009 (blue dotted line) for the selected set of plasma parameters α = 0.4, k_c = 0.0001, σ = 0.01, γ = 0.4, σ_1 = 0.1, υ = 0.01 and constant velocity (u) = 0.01. It is found that the amplitude of ion-acoustic soliton decreases with an increase in k_h however width of the ion-acoustic soliton increases.

In Fig. (4) variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of spectral index of cold superthermal electrons k_c = 0.01 (black solid line), 0.1 (red dashed line) and 0.2 (blue dotted line) for the selected set of plasma parameters α = 0.4, k_h = 0.0015, σ = 0.01, γ = 0.4, σ_1 = 0.1, υ = 0.01 and constant velocity (u) = 0.01. It is found that the amplitude of ion-acoustic soliton decreases with an increase in k_c and width of the ion-acoustic soliton increases.

In Figure (5) variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of γ = 0.3 (black solid line), 0.301 (red dashed line) and 0.302 (blue dotted line) for the selected set of plasma parameters α = 0.4, k_h = 0.0015, σ = 0.01, k_c = 0.0001, υ = 0.01 and constant velocity (u) = 0.01. It is found that the amplitude of ion-acoustic soliton increases with an increase in γ however width of the ion-acoustic soliton decreases.

Conclusions:

In the present study, we have focused to investigate the effect of α, γ, σ, k_c and k_h on characteristics of solitons in unmagnetized plasma. Our main conclusions of the study may be summarized as follow:

(1) We have presented a comprehensive study of small amplitude ion-acoustic soliton in a plasma by deriving nonlinear KdV and mKdV equation using reductive perturbation method and investigated the effect of α, γ, σ, k_c and k_hFor a given set of other plasma parameters as α, γ increases the amplitude of solitary pulses increases and width of soliton decreases.

(2) Amplitude of solitary pulses decreases as σ, k_c and k_h increases but width of soliton increases for a selected set of plasma parameters.

The results of the present investigation may be helpful to understand the nonlinear ion-acoustic solitary waves in space plasma and laboratory plasmas, where two distinct groups of ions and non-Boltzmann distribution electrons are present.

***Lecturer**
Department of Physics
Govt. College, Dholpur (Raj.)
****Lecturer**
Department of Physics
Govt. College, Thanagazi, Alwar (Raj.)

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Figure Captions

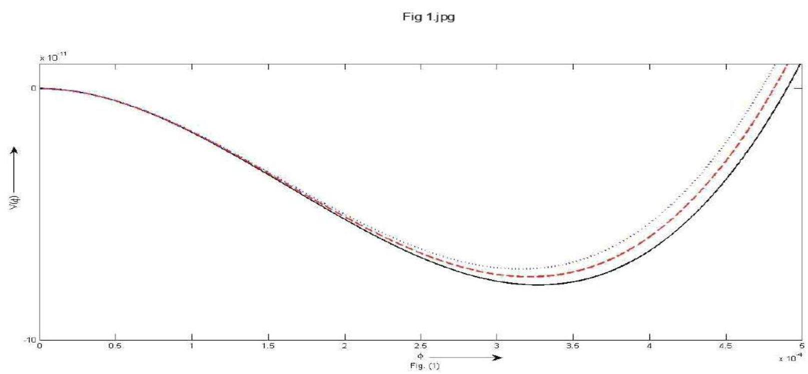

Fig. 1. Variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of σ = 0.5 (black solid line), 0.51 (red dashed line) and 0.52 (blue dotted line), k_c = 0.0001, k_h = 0.0015, α = 0.3, γ = 0.4, υ = 0.01 and constant velocity (u) = 0.01.

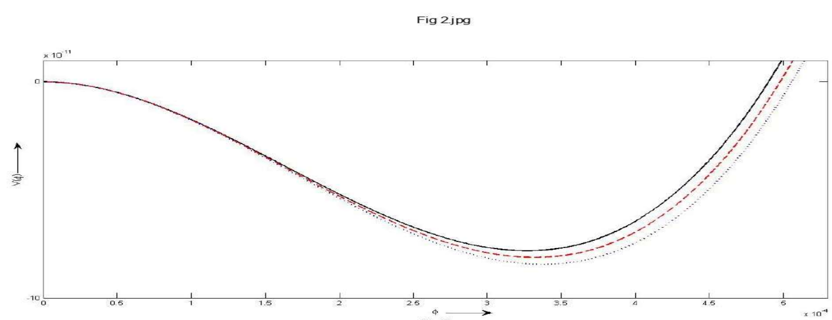

Fig. 2. The variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of α = 0.3 (black solid line), 0.302 (red dashed line) and 0.304 (blue dotted line), k_c = 0.0001, k_h = 0.0015, σ = 0.5, γ = 0.4, υ = 0.01 and constant velocity (u) = 0.01.

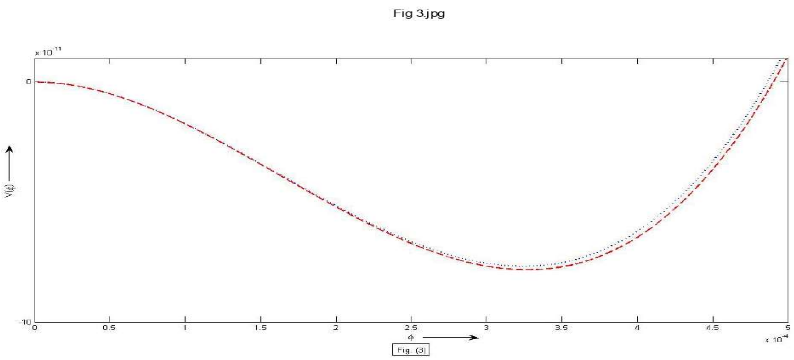

Fig. 3. The variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of k_h = 0.001 (black solid line), 0.005 (red dashed line) and 0.009 (blue dotted line), α = 0.3, k_c = 0.0001, σ = 0.5, γ = 0.4, υ = 0.01 and constant velocity (u) = 0.01.

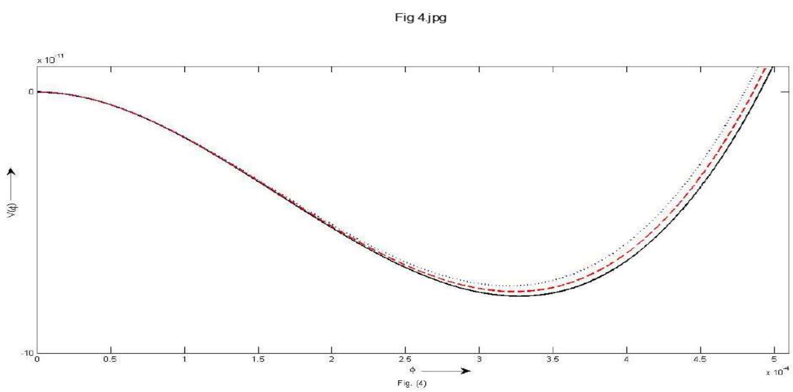

Fig. 4. The variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of k_c = 0.01 (black solid line), 0.5 (red dashed line) and 0.9 (blue dotted line), α = 0.3, k_h = 0.0015, σ = 0.5, γ = 0.4, υ = 0.01 and constant velocity (u) = 0.01.

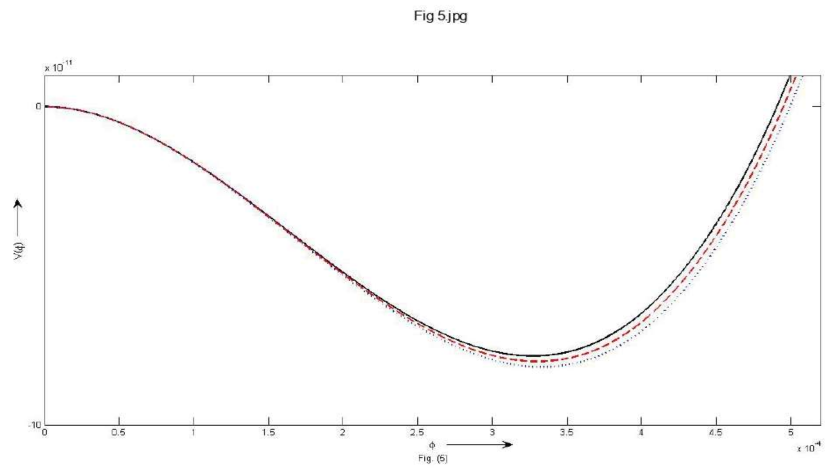

Fig. 5. The variation of the sagdeev potential $V(\phi)$ with potential (ϕ) for different values of γ = 0.3 (black solid line), 0.31 (red dashed line) and 0.32 (blue dotted line), α = 0.3, k_h = 0.0015, σ = 0.5, k_c = 0.0001, υ = 0.01 and constant velocity (u) = 0.01.