A Study on Trigonometric Knowledge: Pedagogical Approaches and Their Impact on Learning Outcomes

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Abstract:

The primary argument of this book is that understanding trigonometry is critical for mathematical and practical success. Trigonometry is useful and important, but pupils have a hard time with it, according to the specialist bibliographies. Hence, this work aims to pinpoint which aspects of this mathematical subfield students struggle with, determine why they struggle, and then provide practical, evidence-based suggestions for improving or at least minimising these weaknesses. To that end, this book aims to provide teachers a framework of pedagogical principles grounded in theory that they may use to guide their students and themselves through trigonometry classes. The student's effort is definitely essential to their education and mental progress, as seen in the specific bibliographies. The proposed idea is based on a number of theoretical principles, including process-to-object transit, theoretical generalisation, and changes to the representational register. A pedagogical experiment, participant study, and a literature review comprise the approach that is used to verify the results.

Keywords: Trigonometry, Didactic Teaching, Semiotic Registers, Generalization, ICT

1. Introduction

Raising the bar for human resources is crucial to expanding our understanding of technology. Investing in mathematics education may assist fulfil the need for trained mathematicians, given the pervasiveness of mathematics in our everyday lives. Students should have developed their capacity for analysis, deductive reasoning, critical thinking, and organisation by the time they finish the course.

According to Báez and Blanco, trigonometry is an integral part of mathematics' systemic structure because its use in both mathematical and non-mathematical contexts highlights aspects of mathematics' ontology, its dual role as a tool and an object in and of itself, and the science's overall organisation. Trigonometry is important in mathematics and science in general, yet it is difficult for both students and teachers, according to research. Common methods of teaching trigonometry fail to provide pupils with a solid foundational understanding of the concept, according to several studies (e.g., Serpe and Frassiaa's). Jerito and Hermita.

Undoubtedly, trigonometry stands as a distinct branch of mathematics. In contrast, the study of metric relations in triangles and their generalisation as functions of real variables need alternative approaches to the classroom teaching of trigonometric ratios. Because of their periodic nature and lack of bijectivity, trigonometric functions have distinct characteristics that distinguish them from other intermediate-level functions, such as logarithmic and exponential functions. One reason trigonometry is so difficult to teach and grasp, according to research, is because traditional teaching techniques don't establish the relationship between triangles and the trigonometric function.

More people should take the time to learn trigonometry as it is useful in many fields outside mathematics. Eratosthenes, for instance, used it to determine the Earth's diameter. Its hyperbolic function-defining usage in civil engineering is in measuring cant angles in curves. The hanging rope issue and related ones may be studied with the help of these functions. The equations that prove the equality: e^i ⼇〽\&–1 prove without a doubt that the systematic structure of mathematics is essential for solving integrals of rational functions in the domain of generalised partial differential equations.

In order for pupils to understand the identities they'll need for different mathematical applications, it's crucial to teach trigonometry by using the connections between the functions. Students may avoid memorisation and deal with material they'll need in the future by seeing how mathematics is built using mathematical instruments. If you believe Doherty, Bellestier, and Rhodes, then...

2. Theoretical Fundament

The foundation of mathematical ontology and epistemology is the non-ostensive character of mathematical things. We are able to adapt mathematical notions via representations in different semiotic registers, say Báez and Blanco. This calls for representations, and more specifically, registers of semiotic representation, as well as transfers between and across them. One of the reasons mathematics education is so difficult on a worldwide scale is that mathematical things do not have a physical counterpart. As pointed out by Duval, the representations indicated above allow us to access mathematical objects and understand the topic.

The transition from procedural to conceptual understanding is also the foundation of this paper's study. Vygotsky argued that rather than being taught a notion in its complete form, pupils should be guided to develop it via the use of algorithms and processes. In this context, Star and Stylianides' definitions of "procedural knowledge" and "conceptual knowledge" are provided. The student's work on different aspects of the work object will be strengthened by representations and changes in semiotic representations.

Students' capacity to conceptually generalise their knowledge is associated with their capacity to change representational registers and to move emphasis from processes to objects. This is due to the fact that, as stated by Karpov and Bransford, scientific ideas are grounded on real-world data instead than being derived from abstract theories.

Development

Understanding "Trigonometric Ratios"

Since mathematical objects are intrinsically non-ostensive, Duval contends that one issue in mathematics education is assisting pupils in internalising mathematical notions via their semiotic materialisations. Because various representations show distinct aspects of the thing under study, you'll need to work it in a variety of ways; When it comes to trigonometric functions, there are two ways to look at them. One is as ratios, defined by the relationships between the sides and angles of a right triangle. The other is as functions, defined by the axis system coordinates. The student is expected to view the latter as a generalisation of the former. However, according to Sampaio and Batista, students often confuse the trigonometric ratios they learnt in high school with the functions they studied before university. Maknun, Rosjanuardi, and Jupri argue that this poses an epistemological obstacle when studying trigonometric functions. Maknun, Rosjanuardi, and Jupri state that no investigation on students' trigonometry comprehension was conducted by Brousseau in light of the epistemic barrier.

If you want your high school pupils to have no trouble understanding trigonometry, you should start with trigonometric ratios. Students should still be motivated to discover new connections using the definitions provided, even with the help of their teachers. So, for example, the Pythagorean theorem leads to various connections, and one of them is the basic equation sin2(x) + cos2(x) = 1.

the sum of the quadratic formulas tan(2x) * 1 * sec(2x) and 1 * cot(2x) * csc(2x) furthermore Using the definitions, students should be able to determine that cos(x)sec(x)=1 and that tan(x)†^\ sin(x) y cot(x)&= cos(x) are equivalent.

Inverse of both sin(x) and cos(x)

The absolute value of sin(x)csc(x) and tan(x)cot(x) is 1. Since it is no longer seen as essential by Classical standards, students shouldn't concentrate on learning trigonometric ratios by completing the task of expressing each function in terms of the remainder.

Students must meticulously determine the values of the trigonometric ratios at important angles in accordance with this idea.

As shown in the images below, all students need to know is that they are an isosceles right triangle with leg one at 45 degrees and that they are an equilateral triangle with side one at 30 and 60 degrees.

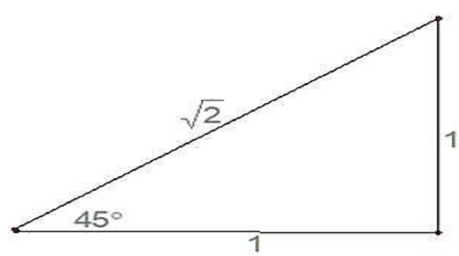

Figure 1. Notable angles 45 degree

Hence, while using substantial values of the trigonometric ratios, it is not essential to depend on tables or memorise them. The answers centred on teaching students to appreciate the tools they have to solve mathematical issues and to see mathematics for what it is: a medium and an object in and of itself, rather than only relying on digital media to obtain information.

Enhancing the degree-radian connection and evaluating students' thinking should also be priorities at this level. Since many students are given radian problems without any prior knowledge, it might be wise to start by providing an explanation of the radian and its mathematical applications: The International System of Units (SI) uses the symbol rad to represent angles; a flat angle with its vertex in the middle of a circle is equivalent to an arc with a length equal to the radius of the circle.

The whole circle will have 2π radians as a result, which is two times r, and so Given that r = radians, we can find that 1°= Theta radians, which is equivalent to 180 degrees.

Complete change

This allows the student to easily transition between several units of measurement. Without active engagement with the material, students will struggle to achieve conceptual comprehension in the procedural job. Trigonometry lessons in the past have focused on memorisation of rules and their applications rather than helping students understand the material by fostering their ability to make connections between different mathematical concepts. Twelve in all, according to Nordlander.

The formula for finding the area of a triangle given its two sides and the angle is one of the results that students can find with the help of their teacher. They can access this formula by constructing the height that corresponds to one of the known sides. Using essentially the same concept, they can also find the law of sines. Furthermore, students may internalise the mathematical systematic framework—the law of cosines—within a little algebraic effort. Returning to the earlier topic of mathematical assistants (software for mathematical work), it is crucial to provide students questions that these assistants cannot solve immediately and that do not need much of their skill.

Students must make an effort to gather the mathematical tools they need to tackle the many interesting real-world challenges that trigonometry may help them with. As one would expect from a trigonometry course, the specific library is replete with solved problems using trigonometric ratios.

Functions in Trigonometry

The most effective tool for teaching pupils about trigonometric functions is the trigonometric circle inside a Cartesian coordinate system. They learn that it's not necessary to depend on memory to obtain the values of the functions in each quadrant and that the terminal side of the angle determines whether the function is positive or negative. Trigonometric functions also have another crucial feature: periodicity. The fact that &π is equal to 3.14 and 180 degrees might be difficult for pupils to grasp at times. Their understanding of the process of converting angles between degrees and radians and vice versa is limited to the procedural level and does not extend to the necessary conceptual level. Therefore, before going on to more complicated ideas, pupils should be helped to get acquainted with the basic ideas.

According to Maknun, Rosjanuardi, and Jupri, there are two things that students need to know before their teachers can help them generalise trigonometric functions from the trigonometric circle: first, that the functions' results can be more broadly applied if the radius is taken as one, and second, that the values of the angles in the circle can be more broadly applied to the coordinate system if used.

Using Geogebra or a comparable math application, students may make the following graph to illustrate how the value of the trigonometric functions remains constant regardless of the size of the circle's radius:

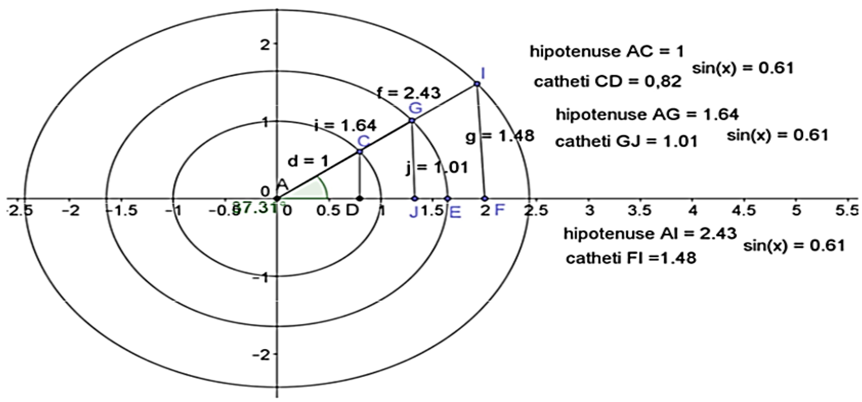

Figure 2. Trigonometric circle

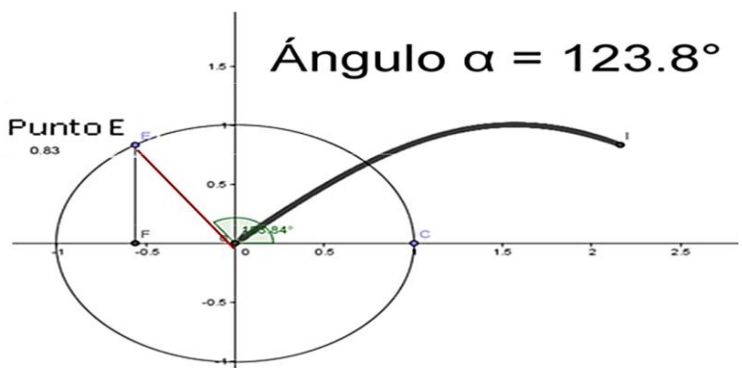

Figure 3. Generation of sine function, angle up to 123 degree

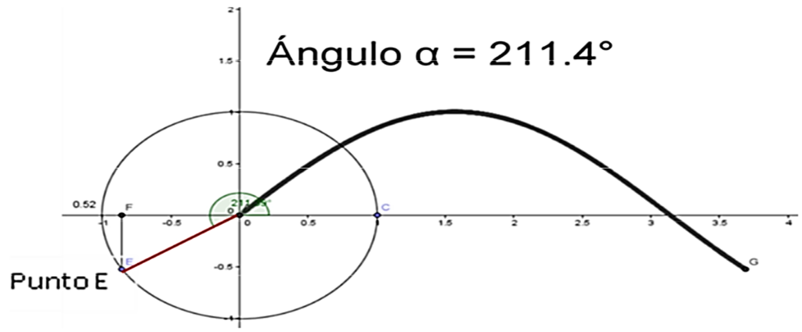

Figure 4. Generation of sine function, angle up to 211 degree

The link between a circle's radius and its matching triangle leg always gives the sine of an angle. Students may have a better grasp of this concept by practicing with the leg to their right and with angles in other quadrants. By doing so, they may determine the value of the function with the associated sign and the fact that the sine of an angle has a constant value independent of the circle's radius.

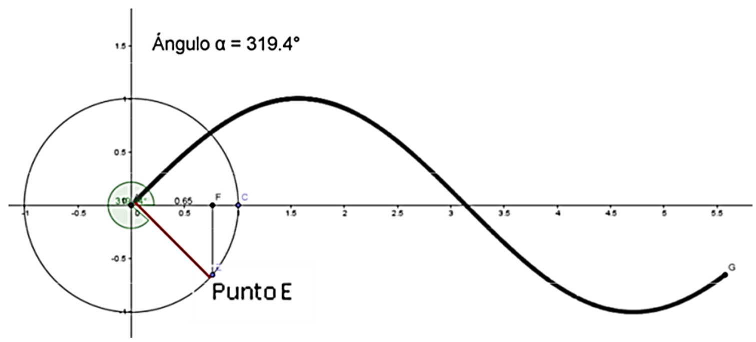

Figure: 5 this coordinate grid

Students may see an example of a dynamic graph that shows how an angle can be expanded inside the trigonometric circle to form the graph of a trigonometric function. The program constructs the sine function by extending the angle with respect to the centre of the circle; you can watch this process in three phases on the graph. To do this, activate the trace of the angle and move the point E, which controls the amplitude, to where you want it to be.

Students will get the greatest knowledge by using the software to see the sine function graph's continuous creation, since it demonstrates all three processes.

Students benefit from understanding many of the characteristics of trigonometric functions, as previously stated. For instance, by seeing and manipulating graphs in dynamic geometry software such as GeoGebra, students may get a better understanding of the functions and their many characteristics. Example: the graphs below illustrate the dissimilarity between sin(nx) and n multiplied by sin(x).

Students may see the effect of the coefficient on the period and the expansion of the ordinate values, respectively, by comparing the red graph of sin(x) with the graphs of sin(2x) and 2sin(x). Students' comprehension is much enhanced when they examine graphs of functions such as y = a× sin(x+b) + c or y = a.

The product of x's cosine with b and c. Researchers Mosese and Ogbonnaya found that students who used graphing software like GeoGebra and adjusted the settings using sliders had a better grasp of graph properties such as amplitude, intersections, periodicity, and range.

The procedural practice is made more efficient and fun for pupils by using dynamic geometry software. The preceding paragraphs laid forth the reasons why pupils must alter their semiotic registers before they can properly understand mathematical things.

Equations and Identities in Trigonometry

Prior to delving into identities and equations, students need to have mastered the concepts of summing angles, half angles, and converting sums into products. In order to solve the identities and equations, students must first obtain the formulae for sin(x+y) and cos(x+y). This will inspire children to take part in this construction while also helping them grasp the idea that mathematical instruments are used to produce mathematics.

Equations in Trigonometry

At first, students have a hard time solving these equations because they restrict their answers to the range of zero to ninety degrees, or even more strictly, zero to three hundred and sixty degrees. The significance of taking into account the range of the trigonometric functions may be better grasped by pupils when they examine all potential solutions to a graph like the one below: Before moving on to solving trigonometric equations, students need have a solid grasp of basic equation concepts, such as how to make an equation equal to zero by substituting numbers. The second item students should be aware of is that in order to solve an equation, one must first identify the one trigonometric function that may be used to represent it.

One possible solution to the equation sin 2(x) & ° cos(x) & ˀ\ 1 may be obtained by substituting sin 2(x) for 1⊥ ∘ cos 2(x).

One minus the cosine of 2(x) minus the cosine of (x) — one -simplifying and factoring Before us is this: The solutions of the equation cos(x) = 0 are connected to the periodicity of the cosine and must be taken into account.

The equation x=&π&± nπ may be expressed for integer values of n beginning with 2.

By solving the equation cos(x) = 1, we get the values x = &±2nπ.

Here we see how the change of variable, a versatile mathematical technique, might be useful in solving a trigonometric equation:

example: The value of tan 2(x) changed is zero since it is equivalent to two times tan(x) plus one. Finding the value of u in the equation leads to the variable tan(x) = 2, and when we return to the starting value of tan(x) = 1, we get the answers x = &π&± nπ.

There are a lot of resources with both solved and unsolved equations in all four of the cited works; teachers can choose good examples to show their students by making sure they don't recycle methods or ignore possibilities.

Emphasising the relevance of solving problems requiring some algebraic effort might help students grasp the hierarchical structure of mathematics. Here's a case in point:

From what source is the fact that 1… ∘ sen(x) 1ـــ° sen(x) = 1 derived?

Because a 2&= a, students need to be reminded that the solution is x = nπ. In particular, it is cos(x) ≠ 1.

An Interest in Trigonometry

As with trigonometric equations, the literature is replete with instances of trigonometric identities. Having said that, pupils should be cognisant of two crucial aspects. To start, you can't move words about inside the equality; instead, you have to focus on one side until you can improve the other, or both. One more option is to get the two variables on the same side of the equality and show that subtracting from zero yields zero. These methods have been discussed extensively in the literature, so we won't repeat ourselves here. However, we will demonstrate a less popular but valuable approach that may be used to confirm the identification of Alonso Blanco.

An example of the procedure is this: Please verify that sec(θ)×csc(θ) is the same as the product of cos(θ) and tang(θ).

The values of each function are shown on the diagram instead of utilising trigonometric transforms. Just as we can get cot(Ϙ) = cos(ϟ) and sin(ϟ) = y by using the trigonometric triangle, we can also obtain that cot(Ϙ) = x and sin(ϟ) = y.

The equation x plus y = xx plus yy, which is equal to x squared plus y squared, may be rewritten as tan(Ϙ) ȑ y, and the correct substitutions can be made on the right side of the equation to show that x plus y is also equal to x squared plus y squared. When the trigonometric circle's relationships are used, the connectivity between the variables is 1.

As per Goel and Elstak [6], the computation of sec(O) × csc(N) proves the identity.

The following example shows how the method improves dramatically with the introduction of powers:Show that cot 2(ΰ)= 1 - sin(ΰ) by repeating the previous stages. When the numerator and denominator are multiplied by 1 plus 1, the result is y2. This is because y2 is equal to 1 plus the square root of csc(β) times sin(β) divided by x2.We may infer that y2equals x2 as x2 + y2 = 1. When we finish the maths, we find that the formula (1 + 2y + 2y) is identical to y plus 2y, where y is an integer between 1 and 2.

The solution to the equation x 2 = 1- y2 allowed us to deduce the identity. Afterwards, we proved that y is the sine of θ by substituting and factoring until we obtained 1- y = 1- sin(α).

Here, considering ways to achieve

When working with powers of trigonometric functions, it is necessary to write the exponent over the function to provide clarity, as seen in the example. instead of the way it appears in some publications, but as cot 2 (x) cos2 because, as one would expect, it is the squared-off angle.

When the identity includes functions evaluated with several parameters, this approach may also be employed; for instance, consider the following: Express yourself clearly:

Add sin(A) to cos(B) plus cos(A).The sum of A and B's tangents is equal to B's sine. continue as the sine of B times half of the tangent of A at B, minus the sine of A minus the cosine of B The representation for each argument is x1 y2 + y1 x2 x1 x2 - y1 y2, which is differentiable, unlike the previous cases.

The identity is x x 1 - y1 y2, which can be obtained by dividing both the numerator and the denominator by x1 x2. Then, we can solve for x2 by dividing both the numerator and the denominator by x1 x2.1, divided by the tangent of A divided by the tangent of B Multiplying x1 by x2 yields the result.

To begin with, it offers a different choice for student assignments. The second benefit is that it helps pupils become or stay technically proficient in algebra. Third, it shows how useful the trigonometry circle is in the real world, which is a huge plus in the workplace.

Students in the "Speciality in teaching basic and upper secondary Mathematics" master's program at APEC University (Action for Education and Culture) examined the ways in which students struggle with trigonometry and designed their lessons accordingly. We next sought the opinions of subject-matter experts via interviews to see how they had used the aforementioned strategies in their own classrooms. Using the methods they had perfected in their master's degrees, every single respondent said they were able to raise their pupils' trigonometry understanding.

Conclusions

Teachers should still go to the appropriate bibliographies for trigonometry examples and exercises, as this website isn't designed to serve that purpose.

We propose that the stated goal—to provide teachers with a set of theoretically grounded principles for teaching trigonometry—can be achieved. Through the use of mathematics software, students can move from the process to the object. Additionally, students can gain a better understanding of mathematical ideas by examining mathematical ontological elements, such as the subject's interrelated systemic structure. This allows them to derive general theoretical conclusions and provide reasons for why teaching trigonometry according to the provided rules is beneficial.

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